Dispersion and Polarization Features of Whistler-Mode Waves in Low-Density Plasma

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The dynamics of energetic electrons in the Earth's radiation belts is largely determined by their interaction with whistler-mode waves which are generated due to electron cyclotron instability. In many regions of the Earth's magnetosphere, in particular, in the region of the radiation belts, the electron plasma frequency ω_p significantly exceeds the electron gyrofrequency ω_c. Under this condition, the dispersion relation for whistler-mode waves, as well as the expressions for the wave refractive index and polarization coefficients as functions of wave frequency ω and wave normal angle θ are explicitly known. Under the same condition, in the theory of whistler waves in the magnetosphere there are a few fundamental and well-known results. These include the following. As has been shown by L.R.O. Storey [1], the group velocity of lower-frequency whistler waves does not deviate from the direction of the ambient magnetic field by an angle greater than 19.47 degrees. For waves with frequencies $\omega < \omega_c/2$, the group velocity is collinear to the ambient magnetic field when the wave normal angle θ is equal to the so-called Gendrin angle $\theta_G = \text{acos}\,(2\omega/\omega_c)$ [2], as well as $\theta = 0$, of course. Another interesting feature of whistler-mode waves under condition $\omega_p^2 \gg \omega_c^2$ consists in that the wave magnetic field is circularly polarized regardless of the frequency and propagation angle. A natural question arises as to how all these results change if the electron plasma frequency is less or significantly less than the electron gyrofrequency. In addition to theoretical interest, this issue is of practical importance in relation to the auroral region of the Earth's magnetosphere, as well as for several regions of Jupiter's magnetosphere, where the condition $\omega_p < \omega_c$ is also satisfied. In the present report, we discuss the dispersion properties of whistler waves under condition $p \equiv \omega_p / \omega_c < 1$. For convenience, we use normalized frequency $f = \omega/\omega_p$ in the relations that follow, valid for $p < 1$.

In the case under discussion, resonance frequency at which the refractive index for whistler waves tends to infinity is equal to $f_{res} = \sqrt{\cos^2\theta/\left(1+p^2\sin^2\theta\right)}$. We consider wave frequencies between lower-hybrid resonance frquency, which is now close to the proton plasma frequency, and f_{res}. In the frequency band $p/(1+p^2) < f < f_{res}$ there are two propagating modes, while for $f < p/(1+p^2)$ there is only one. Maximum frequency $f_{max}(p)$ below which the Gendrin angle exists is shown to be equal to

$$f_{\max}^2 = \frac{4+3p^2 - p\sqrt{8+9p^2}}{8(1+p^2)}. \tag{1}$$

The found analytical expression for the Gendrin angle is rather cumbersome and will be presented in the report. We show that in the case under consideration, the presence of the Gendrin angle is a necessary and sufficient condition for the existence of a maximum, not equal to 90 degrees, angle of deviation of the group velocity from the direction of the ambient magnetic field, the so-called Storey angle. In contrast to high-density plasma, in low-density plasma the polarization of the wave magnetic field is elliptical.

References

[1] L. R. O. Storey, "An investigation of whistling atmospherics", *Phil. Trans. Roy. Soc. London*, **A246**, 1953, pp. 113-141.

[2] R. Gendrin, "Le guidage des whistlers par le champ magnetique", *Planet. Space Sci.,* **5**, 1961, pp. 274-282.