

Excursions and Excess of Power in Stochastic Fields

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Extended Abstract

The crossings and excursions (exceedances) above or below a threshold level are important (conditional) random quantities that require stochastic characterization. The issue is of particular importance in electromagnetic reliability engineering and in testing for electrical fatigue, where the duration of exposure relative to the time constant of the system or environment may be critical to proper operation or malfunction. This is additional to global extreme values of random fields that are smooth (correlated) in space and time.

In this work, we analyze the effect of the energy threshold level on the duration and effective power of its exceedances. The adopted framework is the spectral representation of the sample paths of wide-sense stationary stochastic fields, which was introduced previously and extended here to also include lower tails of χ^2 distributions, relevant to fading phenomena. In particular, the approach allows for establishing a new lower limit on the number of degrees of freedom (independent states) in function of the RMS rate of fluctuation, thus informing the general extreme value (GEV) statistics without recourse to any ad hoc definition for the critical threshold level of autocorrelation in data. Furthermore, an analytic expression for the mean and standard deviation of the time between failures (TBF) is obtained as

$$\langle \text{TBF}^\pm \rangle = \sigma_{\text{TBF}^\pm} = \sqrt{\frac{2\pi\lambda_0^2}{\lambda_2 u_{\text{th}}}} \exp\left(\frac{u_{\text{th}}}{\sigma_U}\right) \left[1 + \mathcal{O}\left(\left(\frac{u_{\text{th}}}{\sigma_U}\right)^{\mp 1/2}\right) \right], \quad \frac{u_{\text{th}}}{\sigma_U} \gg 1, \ll 1$$

with a demonstrated agreement with experimental values obtained in a mode-stirred reverberation chamber (Figure 1(a)). The theory is further extended to obtain the probability distribution of the effective power for narrow- and wideband random fields in a dynamic environment that fluctuates at a relative rate (modulation index) β (Figure 1(b)) [1]. The results form the basis for an analysis of the excess power and energy [2].

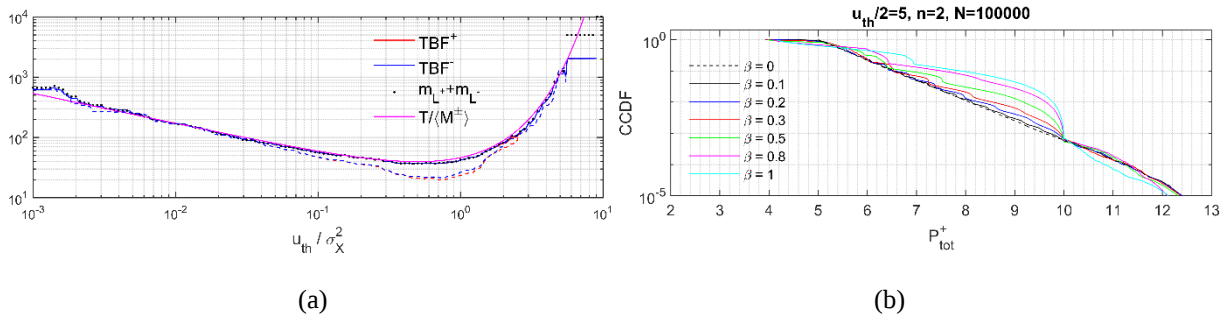


Figure 1. (a) Mean (solid) and standard deviation of time between failures (red) and fades (blue) as a function of normalized threshold level, with comparison to theoretical characteristic (solid magenta). (b) Complementary cumulative distribution of effective Cartesian field power associated with positive excursions, for selected values of relative rate of fluctuation β .

References

- [1] L. R. Arnaut, "Threshold crossings, excursions, and extrema in immunity and fading testing using multistirred reverberation chambers," *IEEE Trans. Electromagn. Compat.*, vol. 62, no. 5, pp. 1638-1650, Oct. 2020.
- [2] L. R. Arnaut, "Excess power, energy, and intensity of stochastic fields in quasi-static and dynamic environments," *IEEE Trans. Electromagn. Compat.*, Dec. 2020, DOI 10.1109/TEM.2020.3039376.