Higher order integrators for relativistic equations of motion for charged particles.

Riku Ozaki* (1) and Takayuki Umeda (1)
(1) Institute for Space Earth Environmental Research Nagoya University e-mail: rozaki@isee.nagoya-u.ac.jp

The purpose of this study is to improve the numerical method for integrating the relativistic equation of motion for charged particles.
The Runge-Kutta integrator (RK4) is a classical method for numerically solving various difference equations. Although RK4 has a fourth order accuracy in time, RK4 does not satisfy the conservation laws in the relativistic motion of charged particles. The Boris integrator [1], which has the second order accuracy in time, has been used conventionally. Although the Boris method satisfies the energy conservation law during the gyro motion of charged particles, the numerical error is larger with a larger relativistic Lorentz factor.

Recently, a new integrator has been developed based on the theoretical solution to the relativistic equation of motion for charged particles [2], which is called as the Umeda integrator. The Umeda integrator conserves the boosted Lorentz factor and gives the exact E-cross-B drift velocity. However, the Umeda integrator has the second order accuracy in time and less accurate than RK4. In this study, we develop advanced integrator with higher-order accuracy.

First, the Umeda integrator has been improved by using multi-step (multi-stage) Runge-Kutta integrators. Second, the Taylor series expansion of the relativistic gyration angle has been made, and then higher-order terms up to the third term have been included. Figure 1 shows the comparison of the numerical errors in the momentum and position obtained by RK4, the Boris integrator, the Umeda integrator, and the present method. It is demonstrated that the present method has the fourth-order accuracy in time and is more accurate than the previous integrators.

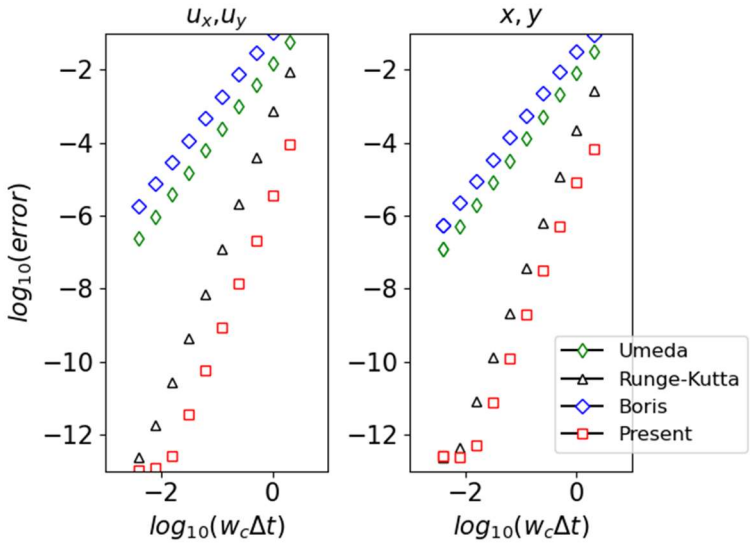

Figure 1. Comparison of error with theoretical value

References

[1] J. P. Boris, “Relativistic plasma simulation-optimization of a hybrid code,” in: *Proceedings of 4th Conference on Numerical Simulation of Plasmas*, 1970, pp.3−67.

[2] T. Umeda, “A new integrator for relativistic E-cross-B motion of charged particles,” *Journal of Computational Physics*,**472**, January 2023, 111694.