URSI GASS 2023, Sapporo, Japan, 19 – 26 August 2023

Bounds, Analyses and Applications of Spectral Asymptotic Decays for 2D Electromagnetic Integral Equations

Ermanno Citraro*(1), Adrien Merlini (2), and Francesco P. Andriulli(1)
(1) Politecnico di Torino, Turin, Italy; e-mail: ermanno.citraro@polito.it; francesco.andriulli@polito.it
(2) IMT Atlantique, Brest, France, adrien.merlini@imt-atlantique.fr

Integral equations represent a computationally convenient way to model electromagnetic scattering by complex objects by leveraging on the link between impinging fields and unknowns on the boundary of the domain. Upon discretization, these equations are represented by a matrix system involving dense matrices that, due to the burden of their computation and direct inversion, must be handled with fast iterative or direct techniques. This has lead to the development of quasi-linear solvers that make use of either fast matrix multiplication algorithm, such as the fast multipole method, or of fast direct strategies. While these often complex techniques can be employed in wide-ranging set of scenarios and for large classes of scatterers, there are well known special cases where particularly simple strategies are extremely effective. One notable example is the 2D modelling of the scattering from a circular target which yields a system that can be easily end efficiently diagonalized by the Fourier basis. Otherwise said, in this very special case the fast Fourier transform allows for the diagonalization of several discrete operators on the circle in quasi-linear complexity. The question that then naturally arises is whether or not such strategies can be extended beyond this specific scenario to a larger class of scatterers. For example one could ask how efficiently the matrix $\mathbf{S}$ resulting from the standard Galerkin discretization of the single layer potential $\mathscr{S}$ can be discretized by a (potentially scaled) Fourier basis for a general geometry. Some studies exists in literature to try to answer to this question often based on the study of the difference between the original operator and an equivalent circulant problem. In this work, we present a more powerful and comprehensive approach that substantially ameliorates the effectiveness of the analysis as well as its practical impact. This is first obtained by focusing on a particular choice of parametrization of the geometry, the arc length function $s(\theta)$. In doing so, two additional orders of asymptotic polynomial decay of the approximated eigenvalues of $\mathscr{S}$ with respect to the complex exponential basis, are gained compared to the state of the art. This is illustrated in Fig. 1, in which the decay of $\mathbf{R}_0^F(s)$ is compared with $\mathbf{R}_0^F(\theta)$, with an ellipse as boundary. In addition, we show that this higher convergence rate can be improved along given spectral directions by analyzing the structure of the successive residuals of $\mathscr{S}$, which is demonstrated in Fig. 1, since the last curve shows an even further improved $\mathscr{O}(|n|^{-6})$ decay. Numerical results confirm the theoretical findings and show the practical impact of the newly developed approaches on the handling of integral equation matrices and their inverses.

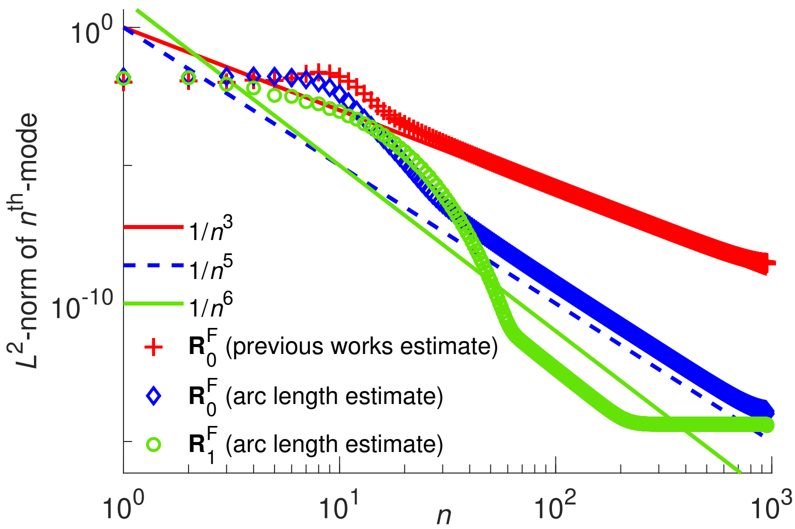

Figure 1. Spectral decays of the single layer potential $\mathscr{S}$ on the ellipse after the first circle subtraction, $\mathbf{R}_0^F$, and that of the successive residual $\mathbf{R}_1^F$. The arc length parametrization $\mathbf{R}_0^F(s)$ adopted in this work provides better normality approximation with respect to the Fourier basis and non trivial bounds for the double layer operator $\mathscr{D}$.