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On a Novel Multi-Kernel Hierarchical Compression Scheme of Boundary Element Matrices

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The Helmholtz equation is used to model acoustic and electromagnetic scattering and radiation problems. The boundary element method (BEM) is one of the most used methods to solve such problems numerically, since it is free from numerical dispersion, only the surface is discretized, and open boundary conditions are naturally incorporated. The BEM has led to the derivation of several distinct formulations, among which the combined field integral equation (CFIE) for acoustics [1] and for electromagnetics [2], are commonly used since they are free from spurious resonances.

The BEM discretization of the CFIE matrices (as well as that of the other formulations) yields dense system matrices which can significantly offset the computational advantages gained from the reduced number of unknowns when compared to other methods, such as the finite element method, that yield large but sparse matrices. This apparent disadvantage can however be alleviated through the usage of hierarchical matrix compression schemes such as the H-matrices [3], that rely on the rank-deficiency of the interaction between well-separated elements of the problem. Typically, most of the time is spent on the setup of the hierarchical matrix, where a low rank approximation for each matrix block of well-separated interactions needs to be computed, and where a significant amount of time is spent on the identification of suitable rows and columns to represent the interaction matrix block. In a CFIE formulation, these columns and rows must be identified for both operators, and, if additional memory savings are desired, compressed together, which, in general, will increase the setup time by around a factor of two.

In this work, we present a new scheme that allows for the compression of different operators using a single hierarchical decomposition which leverages the pseudo-skeleton approximation and a Green's function decomposition for well-separated blocks. An immediate advantage of this approach is that the compression time for complex formulations and the associated memory cost are significantly reduced. The scheme relies on the compression of the Green's function matrix instead of that of the operators themselves. Besides, only a portion of the compressed data is stored in memory, as rows and columns of the compressed blocks are computed on-the-fly. The weak discretization of the operators with basis and test functions are then reconstructed through the usage of sparse mappings, which contain the Gaussian quadrature weights and the evaluated basis functions. The proposed scheme will be presented along with theoretical developments justifying the approach, as well as relevant examples illustrating the effectiveness of the scheme.

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