Euler Classes, Zak Phases, and Surface States of Nodal Photonic Crystals

Haedong Park and Sang Soon Oh*
School of Physics and Astronomy, Cardiff University, Cardiff CF24 3AA, United Kingdom; e-mail: ParkH5@cardiff.ac.uk; ohs2@cardiff.ac.uk

Topological insulators are materials where wave propagations are permitted only on their surfaces. There have been many studies on the topological physics, topological phases, and topological insulators [1]. Most of them exploit the peculiar topology of bands in a band structure or a dispersion relation, for instance, the Dirac [2] and Weyl points [3] and nodal lines [4]. For the recent three years, many progresses have been made on the nodal line crystals using acoustic systems [5] and photonic crystals [6]. However, most of them focus on topological phases themselves rather than their surface states.

In this work, we simulate surface states related with photonic nodal lines in the momentum space. First, we prepare an anisotropic photonic crystal structure called "dielectric double diamond" proposed in Ref. [6]. Our photonic structure consists of $\mathbf{x}$ satisfying $f(\mathbf{x}) > f_c$ where:

$$f(\mathbf{x}) = \sin(X_1 + X_2 + X_3) + \sum_{i=1}^{3} A_i \sin(X_1 + X_2 + X_3 - 2X_i), \tag{1}$$

where $X = [X_1, X_2, X_3] = (2\pi/a)(\boldsymbol{x} - \gamma\boldsymbol{a}/2)$ is a normalized local coordinate defined with the lattice constant $\boldsymbol{a}$. Here, the coefficient γ describes the translation of one diamond from the other along the [1, 1, 1]-direction and A_i describes the deformation in three different directions.

Then, we apply a structural deformation by changing γ to obtain the topological phase transitions of nodal lines. The phase transition is analyzed using the Euler class, an integer topological invariant [5]:

$$\chi_{mn}(\mathcal{D}) = \frac{1}{2\pi}\left[\int_{\mathcal{D}} \mathrm{Eu}^{mn} dk_a dk_b - \oint_{\partial\mathcal{D}} \mathbf{a}(\mathbf{k}) \cdot d\mathbf{k}\right], \tag{2}$$

where $\mathrm{Eu}^{mn}(\mathbf{k}) = \langle \nabla_{\mathbf{k}} u_{\mathbf{k}}^{m} | \times | \nabla_{\mathbf{k}} u_{\mathbf{k}}^{n} \rangle$ is the Euler form and $\mathbf{a}(\mathbf{k}) = \langle u_{\mathbf{k}}^{m} | \nabla_{\mathbf{k}} u_{\mathbf{k}}^{n} \rangle$ is the Euler connection. We also calculate their Zak phases from the following Wilson loop:

$$\mathcal{W}_{pq}(C) = \mathcal{P} \exp\left[i \int_{C} \langle u_{q\boldsymbol{k}} | i\nabla_{\boldsymbol{k}} | u_{p\boldsymbol{k}} \rangle \cdot d\boldsymbol{k}\right]. \tag{3}$$

Finally, we show the surface states of our photonic crystals numerically calculated using COMSOL. The array consists of 16 cells of the double diamond structures, and the finite array is terminated by the perfectly electric conductors. We believe this study will provide a new insight on topologically protected modes in multigap systems and widen the scope of topological physics to electromagnetic waves in three-dimensional photonic crystals.

References

[1] M. Z. Hasan and C. L. Kane, "*Colloquium*: Topological insulators," *Rev. Mod. Phys.*, **82**, November 2010, 3045, doi:10.1103/RevModPhys.82.3045.

[2] S. Wong, M. Saba, O. Hess, S. S. Oh, "Gapless unidirectional photonic transport using all-dielectric kagome lattices," *Phys. Rev. Res.*, **2**, January 2020, 012011(R), doi:10.1103/PhysRevResearch.2.012011.

[3] H. Park, S. Lee, "Double Gyroids for Frequency-Isolated Weyl Points in the Visible Regime and Interference Lithographic Design," *ACS Photonics*, **7**, 6, April 2020, pp. 1577-1585, doi:10.1021/acsphotonics.0c00532.

[4] H. Park, W. Gao, X. Zhang, S. S. Oh, "Nodal lines in momentum space: topological invariants and recent realizations in photonic and other systems," *Nanophotonics*, **11**, 11, February 2022, pp. 2779-2801, doi:10.1515/nanoph-2021-0692.

[5] H. Park, S. Wong, A. Bouhon, R.-J. Slager, S. S. Oh, "Topological phase transitions of non-Abelian charged nodal lines in spring-mass systems," *Phys. Rev. B*, **105**, June 2022, 214108, doi:10.1103/PhysRevB.105.214108.

[6] H. Park, S. Wong, X. Zhang, S. S. Oh, "Non-Abelian Charged Nodal Links in a Dielectric Photonic Crystal," *ACS Photonics*, **8**, 9, August 2021, pp. 2746-2754, doi:10.1021/acsphotonics.1c00876.