URSI GASS 2023, Sapporo, Japan, 19 – 26 August 2023

A review of analytical regularization methods for 2D electromagnetic problems: H-polarization case

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In electromagnetic modelling, integral equations are frequently used to formulate the scattering of waves by two-dimensional perfectly electrically conducting (PEC) surfaces in which one or more apertures are opened. In the H-polarization case, in which the incident magnetic field is parallel to the axis of a cylinder, of arbitrary cross-section, with an aperture, the scattered field is represented by a double layer density. The unknown current distribution is determined by requiring the vanishing of the normal derivative of the total field on the PEC part of the boundary whilst being continuous across the aperture. The resulting integral equation is of first kind with a strongly singular (hypersingular) kernel. Direct discretization produces matrix systems with condition numbers which become unboundedly large as the grid is refined, and no clear convergence to the exact result can be guaranteed. (The condition number provides a precise measure of number of correct significant digits obtainable in the inversion of the system.). Although purely numerical techniques (such as Tikhonov regularization) offer one route to stabilization, the analytical regularization methods discussed in this paper are perhaps more satisfactory than the former which necessarily possess some residual error.

The underlying integral equation may be regularized by analytical inversion of the static part of the hypersingular kernel, a significantly more challenging task than that of the corresponding integral equation arising in E-polarization. Although a theoretical scheme was originally outlined in [1], the complexity of its practical numerical implementation impeded the development of a successful algorithm until quite recently [2]. It is notable for being one of the few algorithms in electromagnetics yielding solutions of guaranteed prescribed accuracy and meeting the criteria laid out in the definitive paper [3] on error estimation in numerical solutions of integral equations arising in electromagnetics. Other related approaches employ inversion of the static part either by using the solution of the Riemann-Hilbert problem [4] or by using the weighted Chebyshev polynomials as expansion functions [5].

Whichever approach is employed, the result is an infinite matrix system of second kind for the Fourier coefficients of the surface distribution. The standard method of solution employs the finite system obtained by truncation to a finite order N; as N increases unboundedly, its solution converges to the exact solution, as desired. In practical cases the convergence is sufficiently rapid that solutions accurate to a prescribed number of significant digits can be guaranteed. Typical results for variously shaped scatterers will be discussed, as well as related problems of transmission in cylindrical waveguides. The approach outlined also provides a practical and stable method for the determination of the complex eigenvalues important for the analysis of resonant features of these structures.

References

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