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A review of analytical regularization methods for 2D electromagnetic problems: E-polarization case

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The concept of regularization is familiar in mathematical modelling of some process and the associated algorithms for its computation. Essentially regularization focuses on ensuring the stability of the numerical processes in the steps of the algorithm. In electromagnetic modelling, integral equations are frequently used to formulate the scattering of waves by two-dimensional perfectly electrically conducting (PEC) surfaces in which one or more apertures are opened. Instability may manifest itself as the underlying grid of the problem is refined and no clear convergence to the exact result is obtained; in another way, small perturbations introduced, either by rounding errors or by other approximations, may produce very large divergences from the unperturbed calculation. In either case, the condition number of the underlying matrix, produced by various discretization methods, which provides a precise measure of number of correct significant digits obtainable in the inversion of the system, will be very large, possibly so large that no reliable numerical results are obtained. Regularization methods become essential. The analytical regularization methods discussed in this paper are perhaps more satisfactory than purely numerical techniques (such as Tikhonov regularization) which necessarily possess some residual error.

In the E-polarization case, in which the incident electric field is parallel to the axis of a cylinder, of arbitrary cross-section, with an aperture, the scattered field is represented by a single layer density. The unknown current distribution is determined by requiring the total field to vanish on the PEC part of the boundary whilst being continuous across the aperture. The resulting integral equation is of first kind with a weakly singular kernel of logarithmic type. Direct discretization produces matrix systems with large condition numbers which become unboundedly large as the grid is refined. The underlying integral equation may be regularized by analytical inversion of the static part of the kernel, effected by the Abel integral transform described in [1, 2]. Other approaches employ inversion of the static part either by using the solution of the Riemann-Hilbert problem [3] or by using the weighted Chebyshev polynomials as expansion functions [4].

Whichever approach is employed, the result is an infinite matrix system of second kind for the Fourier coefficients of the surface distribution. The standard method of solution employs the finite system obtained by truncation to a finite order N; as N increases unboundedly, its solution converges to the exact solution, as desired. In practical cases the convergence is sufficiently rapid that solutions accurate to a prescribed number of significant digits can be guaranteed.

Typical results for single and multiple scatterers will be discussed, as well as related problems of transmission in cylindrical waveguides. The regularization approach outlined also provides a practical and stable method for the determination of the complex eigenvalues important for the analysis of resonant features of these structures.

References

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