



Ray transport and chaos detection in cavities via dynamic indicators

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The propagation of electromagnetic waves in a closed domain with a reflecting boundary amounts, in the eikonal approximation, to the propagation of rays in confined domain, e.g., billiards [1] or waveguides [2]. If the inner medium is uniform the symplectic reflection map provides the polygonal ray paths. We use the linear response theory (LRT) to analyse the stability of any trajectory. In particular, we develop two dynamic indicators from LRT: The Lyapunov and reversibility error. We then present a numerical analysis of: i) a 1D reflection map in a corrugated waveguide [3, 4]; ii) a 2D reflection map in a convex billiard given by a deformed unit circle [5], via computation of the two dynamic indicators. We compare the phase portraits with the colour plots of the Lyapunov and reversibility errors [6] computed in a regular space grid in the ray dynamical phase space. The separation of inot regular and chaotic motion regions is evident for a fixed iteration step, while the presence of sticky orbits requires a refined analysis at the boundary of these regions. Furthermore, we consider the transport of particles and rays within the billiard via Liouville equation similar to the dynamical energy analysis (DEA) method [7]. Given a source of particles or rays within the billiard, the time evolution of the probability density of particles or the energy density of rays is analysed. Such a density can hardly be determined analytically, even for integrable billiards, and a numerical strategy is described. Achieved results are of interest in wave chaos analysis for electromagnetic engineering structures, where deterministic ray chaos often coexist with regular ray trajectories in the realm of propagation environments. Further research is required to extend our formulation to 3D structures, necessary to tackle the analysis of reverberation chambers (RC). Crucially, the RC has been employed in a plethora of electromagnetic engineering applications, including time reversal energy focusing, and EMC testing. For this reason, the chaos quantification, control, and even exploitation of the stochastic RC field has been matter of intense debate over the last decade [8].

References

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