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Wideband Multilayer Impedance Matching by a Minimization Approach

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Abstract

Wideband impedance matching of a purely resistive load by means of a cascade of quarter wavelength long transmission lines is dealt with. The problem is formulated as the minimization of the reflection coefficient modulus inside a fixed frequency range at the variance of the intermediate resistances seen at each line section input. Numerical results are shown and compared with matching techniques based on binomial and Chebyshev coefficient's expansion and on the exponential tapered line.

1 Introduction

Transmission line matching is an old and well-assessed topic, that plays a crucial role in most of the ICT applications of electromagnetic fields. One of the problems in designing a matching network stands in the narrow band, as the matching arises practically at resonance frequencies [1, 2]. It is well recognized that multi-section matching can enlarge the bandwidth because the matching can be achieved by "small steps" that cause a smoothing of the reflection coefficient [3, 4, 5]. Such an approach is based on the theory of small reflections and allows us to ultimately formulate the problem by representing the reflection coefficient by properly chosen coefficients, usually binomial or Chebyshev [3]. Binomial expansion forces the reflection coefficient to be flat inside a prescribed frequency range, whereas Chebyshev expansion forces it to stay below a fixed acceptable maximum value, by accepting as a trade-off an oscillating behavior. As a matter of fact, books and scientific literature report matching schemes based on the above-mentioned techniques where the impedance grows or decays monotonically from the load to the feeding line [3, 6, 7, 8]. However, designing the matching network becomes a hard task when high discontinuities and wideband are desired.

In this contribution, we show that a different approach, based on the minimization of the reflection coefficient modulus, provides better results and, in some cases, a not monotonic solution in terms of the impedance transformation. The proposed approach is tested using numerical examples, deserving the theoretical analysis of the obtained results for future papers. We refer to the quarter wavelength matching scheme, where a resistive load is matched to the feeding transmission line by interposing one or more line sections, each a quarter of a wavelength long at the central working frequency f_c. Moreover, we assume to deal with the ideal lossless case. It is worth noting that losses in the matching network usually help in enlarging the bandwidth at efficiency expenses. However, this is not the case of interest here.

2 Quarter wavelength matching

First, let us briefly recall the classical one-section quarter wavelength matching configuration (Figure 1). A resistive load R_L is connected to the feeding line, whose characteristic impedance is R_0, by means of a quarter wavelength line segment whose characteristic resistance, chosen as $R_x = \sqrt{R_0 R_L}$, guarantees the matching at the frequency f_c. Provided such condition is fulfilled, the reflection coefficient Γ at different frequencies $f = f_c + \Delta f$ can be easily calculated as a function of the relative frequency variation $\eta = \Delta f / f_c$ and of the normalized load $r_L = R_L/R_0$ as

$$\Gamma(\eta, r_L) = \frac{r_L - 1}{r_L + 1 - i2\sqrt{r_L} cot(\frac{\pi}{2}\eta)}. \tag{1}$$

The bandwidth of the matching, defined as the frequency range where the modulus of the reflection coefficient is lower than a prescribed value Γ_{max}, depends on r_L. Note that $\Gamma(\eta, r_L) = \Gamma(\eta, 1/r_L)$, so that what counts in defining the bandwidth is just the ratio between the "starting" and "final" impedance, no matter if the impedance from load to source increases or decreases. In the following, for brevity, we consider $r_L > 1$, keeping in mind that the reciprocal case behaves in the same way. In Figure 2 the modulus of the reflection coefficient for different values of r_L shows a well known behavior: the higher is r_L, the narrower is the bandwidth for a given Γ_{max}, coherently with the trade-off between bandwidth and impedance gap.

A well-established technique to enlarge the bandwidth is to use a cascade of lines, that allows passing from r_L to 1 in small steps so smoothing the frequency dependence and making the reflection coefficient lower than Γ_{max} in a wider range [3, 6]. Figure 3 sketches the transmission lines cascade, where the matching at the central frequency is obtained by inserting N quarter wavelength sections between

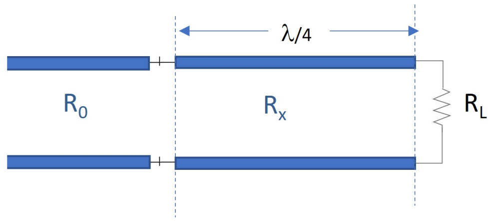

Figure 1. Scheme of matching by means of one quarter wavelength line interposed between load and feeding line.

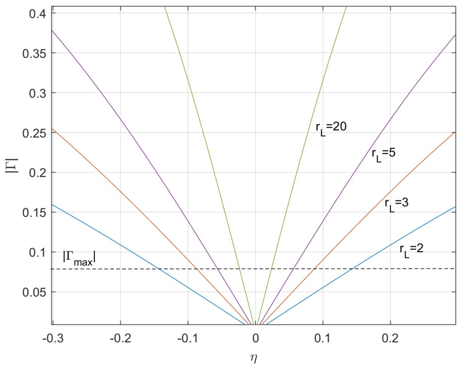

Figure 2. Single-section matching. Modulus of the reflection coefficient as a function of the fractional relative frequency for different values of the normalized load. The maximum acceptable value of the reflection coefficient is exemplified in the dashed line.

the load and the feeding line. All the impedances from now onward are normalized to R_0 and indicated in lowercase. The characteristic impedance of each line is chosen according to the equation $r_{xn} = \sqrt{r_n r_{n+1}}$, $n = 1,\ldots,N$. This choice entails that at the central frequency f_c each line section transforms the load r_{n+1} into r_n, where r_n is the load calculated at the input of the n^{th} line and r_{n+1} is the equivalent load on which the n^{th} line terminates. The r_n are normalized "intermediate" resistances, whose values can be properly chosen to enlarge the bandwidth.

In classical schemes, the intermediate resistances are chosen in a monotonic progression $r_1 < r_2 < \ldots < r_N < r_{N+1}$.

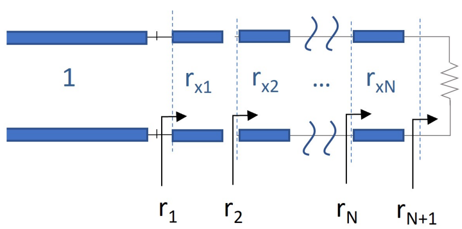

Figure 3. Scheme of impedance matching by a cascade of quarter wavelength lines, each presenting the normalized characteristic impedance r_{xn}. The normalized equivalent impedance at the input section of each line is indicated as r_n.

Note that two of the values are constrained: indeed, $r_1 = 1$ for matching and $r_{N+1} = r_L$. In the proposed scheme, matching is guaranteed at the central frequency whatever values $r_2,\ldots,r_N$ take. This means that there are infinite solutions to the problem, each with a different effect on the bandwidth. .

3 Matching via minimization

Choosing the monotonic progression, according to Chebyshev, Binomial or other expansions, allows to analytically fix some features of the reflection coefficient (bandwidth, maximum level, ripples etc.) but it has not been demonstrated to provide the wider band in the general case. Here, wideband matching is stated as an optimization problem, that consists in minimizing the squared norm of the reflection coefficient for a fixed r_L inside a prescribed interval $\eta \in [-W,W]$:

$$\Phi(r_2,\ldots,r_N) = \sum_{p=1}^{P} \|\Gamma(\eta_p, r_2,\ldots,r_N,r_L)\|^2 \qquad (2)$$

where the η_p are P equispaced values of $\eta \in [-W,W]$. Some points must be underlined before going on. The problem is non-linear, so minimization may present local minima leading to a not optimal solution. Moreover, the same solution's uniqueness is not guaranteed. Despite these aspects, the proposed approach provides a tool to design multiple-step matching and, more importantly, shows that there are cases where the classical monotonic solutions perform worse than the one obtained by minimization.

4 Numerical results and discussion

In the following, minimization is performed by a standard MATLAB routine, the starting point is chosen by fixing $r_n = 1$ for the $N-1$ unknowns $n = 2,\ldots,N$, $P = 1001$, and $W = 0.5$.

Applying the above procedure to the case $N = 2$ provides a result in accordance with the theory: the minimization is reached at the value $r_2 = \sqrt{r_L}$, for different values of r_L and W.

Slight differences between the solutions are obtained in the case $N = 3$. In table 1 we compare the results obtained in the two cases $r_L = 2$ and $r_L = 10$ with those reported in [3]. In order to make the comparison, we calculate the N characteristic resistances r_{xn} corresponding to the $N-1$ intermediate resistance values obtained by minimization (let us stress again that the actual number of unknowns is lower than the number of line sections). The corresponding reflection coefficient is reported in Figure 4. As can be appreciated, the result obtained from minimization is qualitatively similar to that provided by the Chebyshev coefficients. Indeed, we minimized for $W = 0.5$, which is the bandwidth theoretically attainable by fixing $|\Gamma_{max}| = 0.05$ in the Chebyshev approach (see eq. 5.63 in [3]). It can be shown that, at least

Table 1. Comparison of the results for $N = 3$ and $r_L = 2$. Binomial and Chebyshev coefficients (Chebyshev for the case $|\Gamma_{max}| = 0.05$) are taken from reference [3], Tables 5.1 and 5.2.

	r_{x1}	r_{x2}	r_{x3}
Binomial	1.0907	1.4142	1.8377
Chebyshev	1.1475	1.4142	1.7429
Minimization	1.1301	1.4142	1.7698

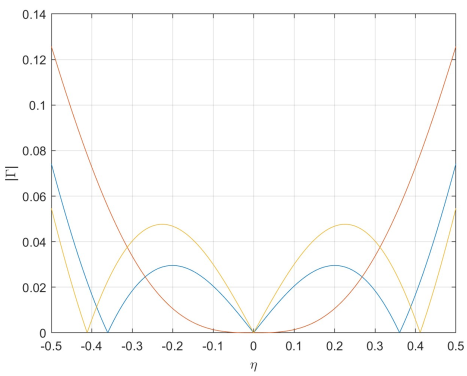

Figure 4. Modulus of the reflection coefficient for $N = 3$ and $r_L = 2$, as a function of the fractional relative frequency, calculated for the impedance values reported in Table 1: binomial (red); Chebyshev (yellow); minimization (blue).

in the case at hand, by properly enlarging the minimization interval $2W$ our approach provides the same results as the Chebyshev expansion in terms of characteristic impedances and reflection coefficient behavior.

More interesting results are obtained increasing both the impedance gap and the number of matching line sections. In the following, we show the case $r_L = 10$ and $N = 10$. This last case is compared with the exponential tapered line, which is a classical benchmark when dealing with a high number of sections, aimed at mimic a continuous taper. In Figure 5 the behavior of the characteristic impedances obtained by minimization is shown. As can be appreciated, it is a non-monotonic function of the index n, different from the discretized exponential taper. The reflection coefficient corresponding to both the intermediate resistances results is shown in Figure 6. The better performance of the non-monotic minimization result is evident.

5 Conclusions

Wideband impedance matching by a minimization approach, which does not rely on the small reflection approximation, has been presented. Numerical results for a small number of stages show that the solutions obtained by the proposed approach are similar to those provided by the technique of Chebyshev coefficients. For a larger number of stages and a high impedance gap, in some cases a non-monotonic growth of the impedance from source to load

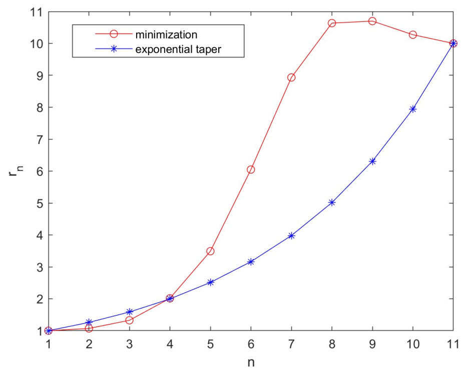

Figure 5. Intermediate normalized resistances obained by minimization in the interval $[-0.5, 0.5]$ for $N = 10$ and $r_L = 10$ compared with the discretized exponential taper.

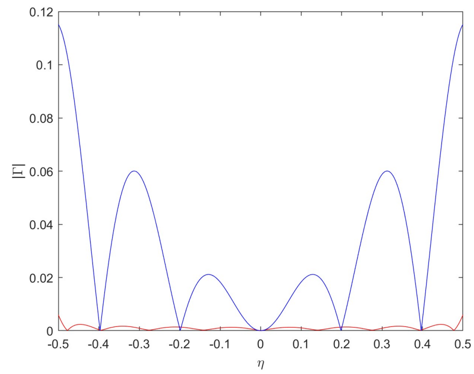

Figure 6. Modulus of the reflection coefficient as a function of the fractional relative frequency, calculated by the impedance values reported in Figure 5 obtained for $N = 10$ and $r_L = 10$: minimization (red); exponential taper (blue).

results. Comparison with a discretized version of the exponential tapered line shows that the obtained solution performs better. The next work will be devoted to investigating theoretically the features of the proposed approach.

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