



Temporal differentiation by neural network designed multilayer metamaterial

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Metamaterials (MTMs) and metasurfaces (MTSs) have gained significant interest due to their unique ability for enhanced control of wave-matter interactions [1]. By introducing spatially or temporally subwavelength/subperiod inhomogeneities, it is possible to carefully obtain effective values of permittivity and permeability [2-6]. This has led to many interesting applications such as invisibility cloaking devices [7], lenses and antennas [8, 9]. Another prominent area that has benefited from the advent of MTMs is analog optical computing [10]. There have been different examples of computational MTMs being used for computing such as switching [11, 12], spatial differentiation [10,13], integration [14] and equation solving [15].

In this work, multilayer MTMs for temporal differentiation (based on alternating layers of ITO and TiO₂) will be presented [16]. It will be shown that for an incident modulated signal with envelope $f(t)$, the envelope of the reflected signal will be given by $df(t)/dt$. Thus, by careful manipulation of the MTM layer thicknesses, the transfer function is made to approximate an appropriate temporal differential operator. Two MTM designs are obtained and used to recover the temporal derivative of three unique incident signals. In all three cases, a mean squared error between the observed and theoretically expected reflection spectra of $\sim 4 \times 10^{-4}$ was calculated. These results have important implications for fast and efficient wave-based computations of mathematical operations such as temporal differentiation, integration and equation solving.

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