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Calderón's preconditioning for the EFIE using collocation and the isogeometric BEM

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The electric field integral equation (EFIE) is one of simpl formulations of boundary integral equations for Maxwell's equations. It is widely known that the discretised EFIE is ill-conditioned, namely iteration methods to solve the discretised EFIE require a lot of iteration numbers, which results in much computational time. One of effective solutions to this problem is the Calderón preconditioning [1]. The Calderón preconditioning constructs a preconditioner based on the so-called Calderón formula, which implies that the square of the electric field integral operator (EFIO) is well-conditioned. Unfortunately simple discretisation of the squared EFIO is known to break down. Hence, in many previous studies of the Calderón preconditioning for the EFIE, the dual basis function is used in addition to the standard basis function, such as the Buffa-Christiansen (BC) basis function [2] and the Rao-Wilton-Glisson basis function. Although the Calderón preconditioning can be successfully implemented with the BC and RWG functions, discretisation of the EFIO with the BC basis function takes much more computational time than that with the RWG function. On the other hand our group has found that the dual basis function for the roof-top basis function defined on a quadrilateral mesh is also the roof-top basis function [3].

Recently our group has proposed the discretisation of the EFIE with collocation [4] using the isogeometric analysis, which utilises the B-spline function for expanding unknown surface currents as well as for representing the geometry of scatterers. In a typical discretisation of the EFIE, namely the use of triangular mesh for approximating boudaries and the RWG function, discretisation based on collocation cannot be formulated since the hyper-singular integral diverges on edges of the triangular mesh. We showed that the collocation can be applied to the EFIE with the help of isogeometric analysis, which holds smoothness of boundaries.

In this presentation, we propose an implementation of the Calderón preconditioning for the EFIE discretised with collocation and isogeometric analysis. In particular we show that the dual basis function in the isogeometric analysis is an expansion of that of the roof-top basis function. Furthermore the proposed dual basis function can be computed without incresing computational time.

References

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