Microwave Antenna Framework Radiating Orbital Angular Momentum Modes based on Spherical Harmonics

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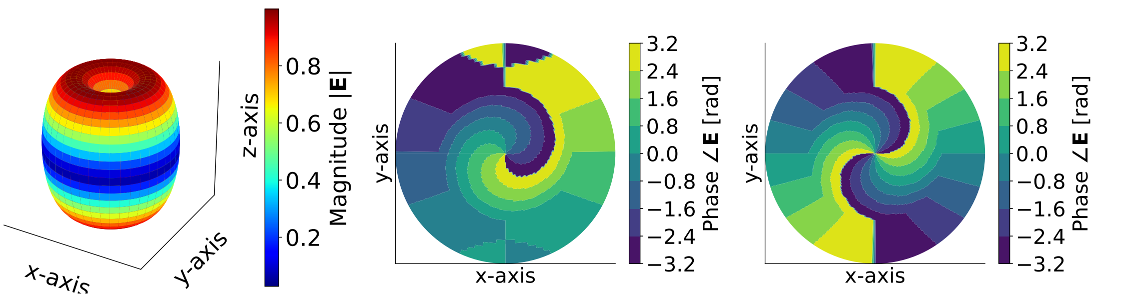
Photons composing electromagnetic waves can be radiated with and carry both linear and angular momentum. Angular momentum can be divided into spin angular momentum and orbital angular momentum (OAM). In particular, OAM is related to the beam vorticity and phase singularity, which can provide unlimited orthogonal modes and offer an additional degree of freedom in communication systems [1]. In this direction, spherical harmonics are functions describing the angular distribution of photons with a particular angular frequency.

Accordingly, in this work we propose spherical harmonics to model the electromagnetic fields of photons in terms of angular momentum states. Accordingly, each spherical harmonic corresponds to a specific angular momentum state, and the coefficients of the expansion give the probability amplitude of the photon being in that state. Therefore, here we derive the far-field radiated electric field $\mathbf{E}$, radiation pattern and wavefronts shown in Figure 1, of different OAM modes being proportional to the spherical harmonics $Y_{m,n}$ with $m = |l| + 1$ and $n = l + \mathrm{sgn}(l)$ as

$$\mathbf{E}(\theta,\phi) \propto Y_{m,n}(\theta,\phi) \propto (-1)^{|l|} e^{j\phi n} e^{jl\frac{\pi}{2}} \sin\theta^{|l|} \left[e^{-\mathrm{sgn}(l) j\frac{\pi}{2}} \cos\theta\hat{\theta} + \hat{\phi} \right], \quad (1)$$

where l is the OAM topological number, θ and ϕ ($\hat{\theta}$ and $\hat{\phi}$) are the azimuthal and polar angles (unit vectors), respectively, in spherical coordinates, and $\mathrm{sgn}(l)$ is the unit-value-operator corresponding to the sign of l.



(a) **(b)** **(c)**

Figure 1: (a) Common radiation pattern shape of spherical harmonics carrying OAM, and wavefront phase pattern along the positive z-axis for (a) $l = 1$ and (b) $l = 2$ OAM modes (engineering convention).

Correspondingly, spherical harmonic modes can be generated with multipole microwave antenna radiating structures, as shown in [2, 3]. In this regards, following the work explored here, it is of relevant interest to theoretically derive and experimentally evaluate an antenna framework modelling the generation of microwave radiated photons carrying OAM modes, based on multipole moment expansion of spherical harmonic radiation sources.

References

[1] R. Chen, H. Zhou, M. Moretti, X. Wang, and J. Li, "Orbital Angular Momentum Waves," *IEEE Comm. Surv. & Tut.*, vol. 22, pp. 840–868, 2020.

[2] M. H. Hassan, B. Sievert, A. Rennings, and D. Erni, "Generation of Vortex Waves Using Crossed Dipole Antennas," in *2021 15th European Conference on Antennas and Propagation (EuCAP)*, Mar. 2021, pp. 1–3.

[3] J. Xu, K. Bi, R. Zhang, Y. Hao, C. Lan, K. D. McDonald-Maier, X. Zhai, Z. Zhang, and S. Huang, "A Small-Divergence-Angle Orbital Angular Momentum Metasurface Antenna," *Research*, Nov. 2019.