# Arbitrary order analogue differentiation using transmission line techniques

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In recent years, analogue computing has seen a experienced an increase in scientific interest in part due to the introduction of "computational metamaterials" in 2014 by Silva *et al*[1]. Since then, various groups have reported analogue processors capable of performing mathematical operation onto the wavefront of electromagnetic or acoustic signals[2]–[5]. Different techniques have been employed in order to achieve this goal including Bragg gratings[6], plasmonic structures[7], multilayered media[1, 8], metamaterials and metasurfaces[9]. These devices have been demonstrated performing fundamental computing operations such as integration, differentiation and convolution as well as solving ordinary differential equations[10], partial differential equations[11] and integral equations[9]. Within the realm of analogue computing differentiation is an especially important operation as it enables edge detection[7], an early step in any image/signal recognition algorithm.

In this work, we present a simple method of calculating temporal derivatives of analogue incident signals using transmission line techniques[12]. We present the fundamental physical principles behind the operation of the proposed structures, demonstrating how they may be used to achieve arbitrary order differentiation, including fractional differentiation. These results are corroborated via time domain simulations. All numerical results are in excellent agreement with the theoretically calculated values. We envision that this work may lead to the further development of time domain waveguide network based analogue processors, which may see application in scenarios where specific computational tasks must be performed repeatedly onto large datasets.

## References

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