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Spatial Aliasing and Accessible Wave Vectors for an Arbitrary Constellation of Spacecraft

Gérard M. Chanteur
LPP, Sorbonne Université , Paris, France; e-mail: gerard.chanteur@lpp.polytechnique.fr

The CLUSTER mission launched in year 2000 opened the era of three dimensional study of wave fields in space physics. In this perspective, the problem of spatial aliasing of the wave spectrum was investigated in the early 90's [1, 2]. Further developments were detailed in chapters 3 and 14 of [3], and later in chapters 5 and 6 of [4]. Two monochromatic plane waves cannot be distinguished by a cluster of four spacecraft when their wave vectors $\vec{K}_1$ and $\vec{K}_2$ are related by the following aliasing condition [1] involving the reciprocal vectors $\vec{k}_\nu$ of the tetrahedron :

$$\vec{K}_2 - \vec{K}_1 = 2\pi \sum_{\nu=1}^{4} n_\nu \vec{k}_\nu \text{ where } \vec{k}_4 = \frac{\vec{r}_{12} \times \vec{r}_{13}}{\vec{r}_{14}.(\vec{r}_{12} \times \vec{r}_{13})} \text{ with cyclic permutations of the indices} \qquad (1)$$

The first analysis of CLUSTER data by K- filtering [5] discussed thoroughly the aliasing problem encountered when analyzing waves propagating inside the magnetosheath, and demonstrated a way to identify aliased energy peaks appearing inside the fondamental cell of accessible wave vectors. Several analyses followed of other CLUSTER data and more recently of MMS data. The success of CLUSTER and MMS missions promoted new projects involving more than four spacecraft. Of course, different tetrahedra could be extracted from a given constellation with $N > 4$, nevertheless some unexpected benefices can be obtained by a generalization of reciprocal vectors to $N > 4$. Standard reciprocal vectors, as defined in equation (1), are generalized by introducing N positive weights w_ν, computing the position tensor $\mathbf{R}$ and it inverse $\mathbf{Q}$ accordingly to :

$$\mathbf{R} = \sum_{\nu=1}^{N} w_\nu \vec{r}_\nu \vec{r}_\nu^{\,T}, \text{ with } \textstyle\sum_{\nu=1}^{N} w_\nu \vec{r}_\nu = 0, \text{ and } \vec{q}_\nu = \mathbf{Q}\vec{r}_\nu$$

The generalized aliasing condition is written :

$$\vec{K}_2 - \vec{K}_1 = 2\pi \sum_{\nu=1}^{N} w_\nu n_\nu \vec{q}_\nu \qquad (2)$$

Aliases of the origin of k-space are defined from equation (2), that allows to specify the fundamental cell corresponding to the weighted constellation. It will be shown that weighting the constellation changes the fondamental cell, possibly opening new ways to identify aliases inside it.

References

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