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Characteristic Modes and Energy Modes using Impedance Operator from Mie Series

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Characteristic modes (CMs) of arbitrarily shaped perfectly conducting (PEC) objects were originally defined using scattered fields [1] and shortly thereafter in [2] using the impedance operator of the electric field integral equation (EFIE). Most commercial and academic implementations are today based on [2], while [1] gives a more direct connection to Mie scattering [3]. In this presentation, we revisit the modes of PEC spheres by deriving an explicit expression for the impedance operator of a PEC sphere using Mie series. This gives the previously well know analytical CMs of a sphere as well as a new analytical expression for the energy modes [4] (EMs) of a sphere.

Figure 1 shows both the CM and EM eigenvalues for a PEC sphere as function of the size parameter $x = ka$, where k is the wavenumber and a the radius. The shown modes are the 4 lowest TE_n and TM_n modes, which actually is 48 modes in total due to degeneracy, and the numerical agreement with the EFIE-based solution is excellent.

The analytical CMs and EMs of a sphere provide a useful benchmark for testing purposes and this result also show how large spheres are needed to get negative EM eigenvalues, i.e., negative estimates for the stored energy [5].

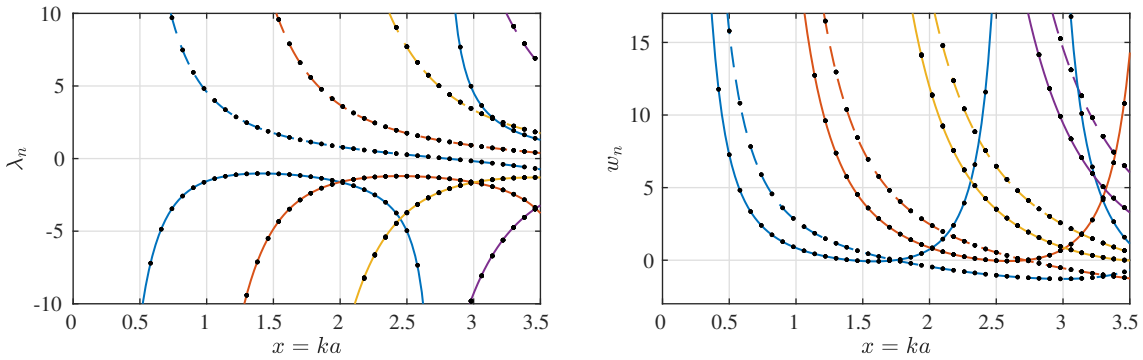

Figure 1. CM eigenvalues λ_n (left) and EM eigenvalues w_n (right) as function of the size parameter $x = ka$ of a PEC sphere. Analytical results with colored lines (TM_n solid, TE_n dashed, different colors for each n) and EFIE-based results with black dots.

References

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