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A theoretical comparison of NavIC and GPS RAIM performance

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Abstract

Receiver Autonomous Integrity Monitoring (RAIM) is a critical capability for a Global Navigation Satellite System (GNSS) or Regional Navigation Satellite System (RNSS) receiver, designed for safety-critical applications. It is of extreme importance to assess the RAIM performance corresponding to a particular navigation constellation to understand the said constellation's independent integrity capability. The Horizontal Protection Level (HPL) is one of the key indicators of integrity which directly depends on the geometric distribution of navigation satellites in space. In this article, the HPL of NavIC and GPS is contrasted to assess the theoretical RAIM capability of the NavIC system.

Keywords: NavIC, GPS, RAIM, HPL

1 Introduction

Global Navigation Satellite System (GNSS) has become a critical space-based infrastructure supporting various critical infrastructures ranging from transport to power grid. Recently Indian regional navigation satellite system, Navigation using Indian Constellation (NavIC) has become fully operational and it is expected that many critical applications within India will rely on NavIC services. The ground track of the NavIC constellation comprising 3 geostationary satellites and 4 inclined geosynchronous satellites are shown in figure 1.

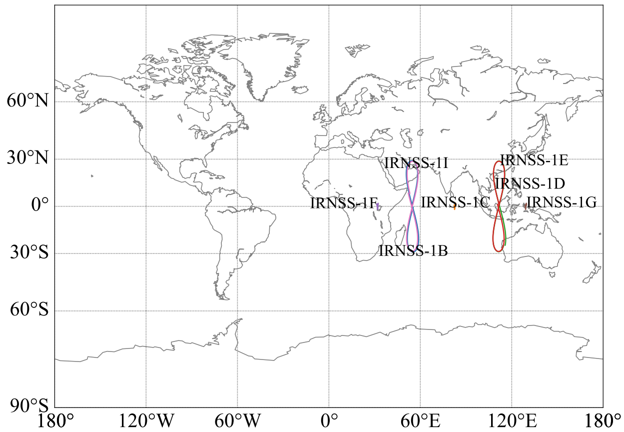

Figure 1. NavIC constellation

Integrity information, i.e. providing a timely warning when a fault occurs in a GNSS/RNSS must be produced by a GNSS/RNSS receiver which is in use for safety-critical applications, for example, civil aviation. Stringent integrity requirement must be met by the GNSS/RNSS receiver before it can be used for safety-critical applications. At many instances, the receiver is capable of providing the integrity information independently using Receiver Autonomous Integrity Monitoring (RAIM).

The integrity performance of Global Positioning System (GPS) has been well documented in [1–3]. Multi-constellation RAIM has also been assessed in [4, 5]. Recently Kalman Filter based RAIM for multi-constellation navigation using GPS and NavIC is also explored [6]. However, it is of importance to explore the NavIC integrity performance separately and benchmark with the GPS to assess the NavIC performance in scenarios when other GNSS services are not available, as well as for further development of the constellation itself.

In this article, the Horizontal Protection Level (HPL), a key indicator of integrity for NavIC and GPS are compared using numerical analysis. It is observed that although the calculated HPL provided by NavIC is significantly higher than that of GPS, it is well within the internationally approved specification of LNAV/VNAV.

2 Receiver Autonomous Integrity Monitoring

A GNSS or RNSS receiver that can be used for safety-critical applications, must be able to produce a timely warning when the satellite system cannot be used for navigation [7]. RAIM algorithm provides the integrity enhancement mechanism to a GNSS/RNSS receiver without any support from additional external systems.

RAIM algorithm provides a horizontal as well as vertical boundary centred to the true position of the receiver within which the position error is contained, based on the geometric distribution of satellites with respect to the receiver, a desired false alarm rate and a desired missed fault detection rate [3]. The horizontal boundary is called the HPL and is an important indicator of integrity.

There are various RAIM algorithm implementations available which are generally driven by the position estimation algorithm that is in use. However, the basic RAIM algorithm [1,3,8] which is applicable for snap-shot Least Square Estimation (LSE) based navigation, is an elegant means of assessing the integrity of a navigation constellation. In the subsequent part of this section, the computation of HPL in the basic RAIM algorithm is described and how HPL is affected by the satellite distribution is explained.

2.1 Satellite geometry and HPL

The linearised GNSS/RNSS pseudo-range measurement equation can be written as [7]

$$\Delta\boldsymbol{\rho} = \boldsymbol{H}\Delta\boldsymbol{X} + \boldsymbol{\epsilon} \quad (1)$$

where, $\Delta\boldsymbol{\rho}$ is the pseudo-range residue vector, $\boldsymbol{H} = \frac{\partial\rho}{\partial\boldsymbol{X}}$, $\boldsymbol{X}$ is the vector containing the receiver position and the receiver clock bias and $\boldsymbol{\epsilon}$ is a zero mean white noise vector.

The estimated error in $\boldsymbol{X}$ using the LSE is

$$\begin{aligned}\widehat{\Delta\boldsymbol{X}} &= (\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{H})^{-1}\boldsymbol{H}^{\boldsymbol{T}}\Delta\boldsymbol{\rho} \\ &= \Delta\boldsymbol{X} + (\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{H})^{-1}\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{\epsilon} \quad (2)\end{aligned}$$

Error in estimation $\boldsymbol{d}$ is

$$\begin{aligned}\boldsymbol{d} &= \widehat{\Delta\boldsymbol{X}} - \Delta\boldsymbol{X} \\ &= (\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{H})^{-1}\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{\epsilon} \quad (3)\end{aligned}$$

Here, $\Delta\boldsymbol{X}$ is the true error. Now, the Linearised measurement equation in presence of fault in the system is [9]

$$\Delta\boldsymbol{\rho} = \boldsymbol{H}\Delta\boldsymbol{X} + \boldsymbol{f} + \boldsymbol{\epsilon} \quad (4)$$

Here, $\boldsymbol{f}$ is the fault vector. The estimation error in presence of fault is

$$\boldsymbol{d} = (\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{H})^{-1}\boldsymbol{H}^{\boldsymbol{T}}(\boldsymbol{f} + \boldsymbol{\epsilon}) \quad (5)$$

For fault detection in RAIM using parity method, the parity vector is defined as [3]

$$\bar{\boldsymbol{r}} = \boldsymbol{S}\Delta\boldsymbol{\rho} \quad (6)$$

Here, $\boldsymbol{S} = \boldsymbol{I} - \boldsymbol{H}\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{H})^{-1}\boldsymbol{H}^{\boldsymbol{T}}$. For some range bias or fault b_i in the i^{th} satellite, considering $\epsilon_i \sim \mathcal{N}(0, \sigma_i)$, the parity vector $\bar{\boldsymbol{r}}_i$ can be written as

$$\bar{\boldsymbol{r}}_i = \begin{bmatrix} S_{1i} \\ S_{2i} \\ \vdots \\ S_{(n-4)i} \end{bmatrix} b_i \quad (7)$$

The test statistic is defined as

$$\begin{aligned}|\bar{\boldsymbol{r}}_i| &= \sqrt{\bar{\boldsymbol{r}}_i^T\bar{\boldsymbol{r}}_i} \quad (8) \\ &= b_i\sqrt{S_{1i}^2 + S_{2i}^2 + \ldots + S_{(n-4)i}^2} \quad (9)\end{aligned}$$

It should be noted that $\bar{\boldsymbol{r}}_i^T\bar{\boldsymbol{r}}_i$ follows the χ^2 distribution because the distribution of the pseudo-range residue is ideally Gaussian. Hence, for a given probability of false alarm P_{fa}, a detection threshold T_{DT} for the test statistic can be calculated from the χ^2 probability density function. If the test statistic is less than T_{DT}, then the system integrity can be declared and conversely, if the test statistic is higher than T_{DT}, then a fault detection alert is provided.

Now, by defining $(\boldsymbol{H}^{\boldsymbol{T}}\boldsymbol{H})^{-1}\boldsymbol{H}^{\boldsymbol{T}} \equiv \boldsymbol{G}$, the estimation error $\boldsymbol{d_i}$ for bias or fault b_i

$$\boldsymbol{d}_i = \begin{bmatrix} G_{1i} \\ G_{2i} \\ G_{3i} \\ G_{4i} \end{bmatrix} b_i \quad (10)$$

From equation 9, one can write

$$\begin{bmatrix} d_{1i} \\ d_{2i} \\ d_{3i} \\ d_{4i} \end{bmatrix} = \begin{bmatrix} G_{1i} \\ G_{2i} \\ G_{3i} \\ G_{4i} \end{bmatrix} \frac{|\bar{\boldsymbol{r}}_i|}{\sqrt{S_{1i}^2 + S_{2i}^2 + \ldots + S_{(n-4)i}^2}} \quad (11)$$

Then the horizontal error

$$\sqrt{d_{1i}^2 + d_{2i}^2} = \frac{\sqrt{G_{1i}^2 + G_{2i}^2}}{\sqrt{S_{1i}^2 + S_{2i}^2 + \ldots + S_{(n-4)i}^2}}|\bar{\boldsymbol{r}}_i| \quad (12)$$

$\frac{\sqrt{G_{1i}^2 + G_{2i}^2}}{\sqrt{S_{1i}^2 + S_{2i}^2 + \ldots + S_{(n-4)i}^2}}$ can be viewed as the slope of the horizontal error vs. the test statistic line. The HPL is calculated using the maximum slope $slope_{max}$ possible, at the time of observation. Considering the Gaussian distribution of the measurement noise, the HPL can be calculated as [10]

$$HPL = slope_{max}T_{DT} + k\left(1 - \frac{P_{MD}}{2}\right)HDOP\sigma \quad (13)$$

Where, $k(.)$ is the inverse cumulative distribution function for Gaussian PDF, P_{MD} is the specified missed detection probability, $HDOP$ is the Horizontal Dilution of Precision at the time of observation and σ is the standard deviation of the pseudo-range measurement.

3 Simulation

Since the HPL depends on the geometric distribution of satellites with respect to the receiver, a numerical simulation is sufficient for the RAIM performance comparison of the NavIC and GPS constellations. Both the constellations were simulated using the System Tool Kit (STK) software. The azimuth, elevation and range of each satellite were recorded from specific locations at some of the major cites in India, from the simulation scenario to obtain geometric distribution information of each satellite constellation. The above-mentioned data were recorded for 24 hours at 15 minutes interval. This information is sufficient to calculate $\boldsymbol{H}$ matrix and in turn subsequent calculation of the HPL.

4 Results

The HPL is calculated using the formulation described in section 2.1 for NavIC and GPS for 24 hours from fixed locations at 7 arbitrarily selected cities in India using the simulated data mentioned in section 3.

Before discussing the main result, it is necessary to explain the effect of the number of visible satellites for a particular constellation on the distribution of the test statistic. As mentioned earlier, the test statistic follows χ^2 distribution and depends on the degree of freedom of the test statistic. The degree of freedom is $n_x - 4$, where n_x is the number of satellite visible for a particular constellation.

Note that, in the Indian subcontinent, all 7 satellites of the NavIC constellation are visible and hence for the NavIC constellation the degree of freedom of the test statistic is 3. However, for the GPS constellation, the degree of freedom will vary depending on the number of satellites visible at the region at a particular time. Figure 2 shows the visibility of both the NavIC and GPS constellations during the simulation time interval from New Delhi location.

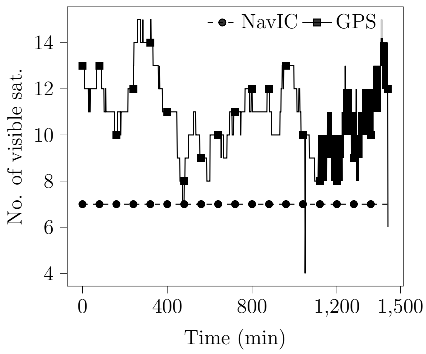

Figure 2. Satellite visibility

The normalised test statistic distribution for the NavIC and GPS from New Delhi at the beginning of the simulation is shown in figure 3. For a false alarm probability $P_{fa} = 10^{-5}$, the threshold of the normalised test statistic i.e. residue square sum for both the NavIC and the GPS are also indicated. Due to the difference in the degree of freedom of the threshold for the NavIC is less than that of the GPS.

The normalised threshold converted to range considering a conservative User Range Accuracy (URA) of 10 m for both the constellations, at a fixed location at New Delhi for 24 hours is shown in figure 4. This result is in agreement with the previous observation that the integrity alarm threshold for the NavIC is less than that of the GPS for a given false alarm rate.

The variation of maximum slope of the horizontal error vs. test statistic line for the NavIC and GPS at New Delhi location is shown in figure 5. It can be observed that the slope

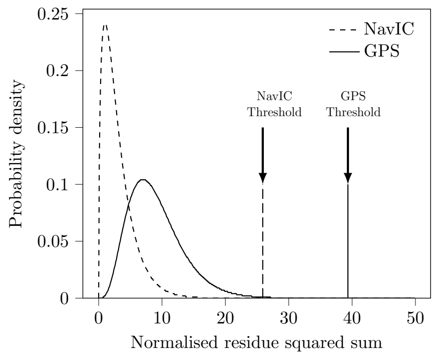

Figure 3. Density

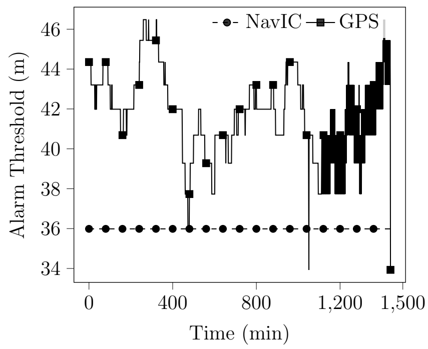

Figure 4. Alert threshold comparison

for NavIC is significantly higher than that of the GPS except for one particular instance, where the number of visible GPS satellites was 4.

The variation of the HPL calculated using equation 13 over the 24 hours of simulation period at New Delhi location for both the NavIC and GPS is shown in figure 6.

It can be observed that the HPL provided by the NavIC is significantly higher than the HPL provided by GPS, mostly due to the geometric distribution of NavIC satellites. However, the NavIC HPL is well within the International HPL requirement of 556 m [11] for non-precision approach and LNAV/VNAV. In table 1 the mean, maximum and minimum HPL of the NavIC and GPS during the 24 hours of

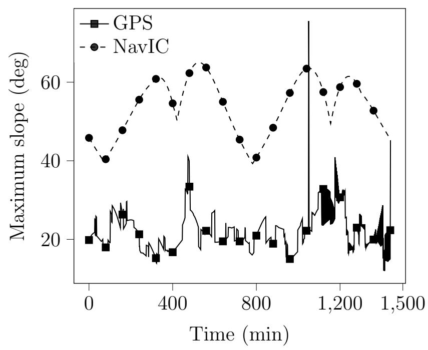

Figure 5. Maximum slope

Table 1. HPL comparison at cities in India

	GPS			NavIC		
City	Mean(m)	Max(m)	Min(m)	Mean(m)	Max(m)	Min(m)
New Delhi	43.086	125.018	30.253	118.26	147.447	91.511
Ahmedabad	40.896	202.197	28.43	116.72	148.132	87.289
Bengaluru	38.354	181.641	29.116	110.522	144.379	76.566
Bhopal	41.035	61.564	29.748	115.378	146.671	85.769
Imphal	42.353	105.064	31.467	118.779	148.201	90.364
Kochi	37.907	80.919	29.07	109.75	144.116	74.937
Kolkata	41.848	69.057	31.497	115.561	146.165	85.748
Mumbai	40.362	74.433	27.641	114.721	147.511	83.396
Nagpur	40.753	99.381	30.553	114.089	145.939	83.517

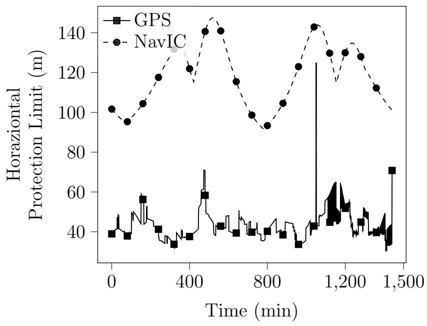

Figure 6. Horizontal Protection Level

simulation interval for various cities in India are shown. The results are in agreement with the observation that the NavIC provided HPL can be well within international HPL standard. However, it is expected that with augmentation using Indian Space-Based Augmentation System (SBAS) GAGAN, a much less HPL can be achieved for NavIC.

5 Conclusion

In this article, the integrity performance of the NavIC and GPS constellation is compared in terms of the HPL. Using STK-based simulation of both the constellations the HPL are calculated from fixed locations at various cities in India for 24 hours. It is observed that the NavIC HPL is significantly less than the HPL provided by the GPS, due to the difference in the geometric distribution of satellites of the mentioned constellations. However, it is also observed that the NavIC HPL is well within the internationally accepted HPL requirement for LNAV/VNAV. It is expected that the HPL for NavIC can be further improved using augmentation using the SBAS.

References

[1] F. van Diggelen and A. Brown, "Mathematical aspects of GPS RAIM," in *Proceedings of 1994 IEEE Position, Location and Navigation Symposium - PLANS'94*, pp. 733–738, Apr. 1994.

[2] T. Walter and P. Enge, "Weighted raim for precision approach," in *PROCEEDINGS OF ION GPS*, vol. 8, pp. 1995–2004, Institute of Navigation, 1995.

[3] R. G. Brown, "A Baseline GPS RAIM Scheme and a Note on the Equivalence of Three RAIM Methods," *Navigation*, vol. 39, pp. 301–316, Sept. 1992.

[4] S. Hewitson, H. K. Lee, and J. Wang, "Localizability analysis for GPS/Galileo Receiver Autonomous Integrity Monitoring," *The Journal of Navigation Royal Institute of Navigation*, vol. 57, pp. 245–259, 2004.

[5] J. Blanch, T. Walter, and P. Enge, "Critical Elements for a Multi-Constellation Advanced RAIM," vol. 60, no. 1, p. 17, 2013.

[6] S. Bhattacharyya and D. Mute, "Kalman filter-based raim for reliable aircraft positioning with gps and navic constellations," *Sensors*, vol. 20, no. 22, p. 6606, 2020.

[7] E. D. Kaplan and C. Hegarty, eds., *Understanding GPS: Principles and Applications*. Artech House Mobile Communications Series, Boston: Artech House, 2nd ed ed., 2006.

[8] B. W. Parkinson and P. Axelrad, "Autonomous GPS Integrity Monitoring Using the Pseudorange Residual," *Navigation*, vol. 35, pp. 255–274, June 1988.

[9] R. Patton and J. Chen, "A Review of Parity Space Approaches to Fault Diagnosis," *IFAC Proceedings Volumes*, vol. 24, pp. 65–81, Sept. 1991.

[10] Y. Jiang and J. Wang, "A New Approach to Calculate the Horizontal Protection Level," *Journal of Navigation*, vol. 69, pp. 57–74, Jan. 2016.

[11] *Global Positioning System Wide Area Augmentation System (WAAS) Performance Standard.* FAA.