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Direct-path Delay Estimation under Closely-spaced Multipath Interference

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Abstract

For direct sequence spread spectrum (DSSS) signal, the minimum mean square error (MMSE) algorithms based on reiterative inverse filters have good delay resolution under closely-spaced multipath interference. However, the limited density of the correlator function dictionary leads to a model error of the filter. Under the inexact model, the performance of delay estimation decreases. To address this problem, we add a group of correction parameters into the dictionary matrix and estimate them in the reiterative filtering. Based on the sparsity of correction parameters, a threshold decision is adopted to sift the parameters that need to be estimated by maximum likelihood (ML) search. The parameters that fail to pass the threshold are set to zero. Simulation results show that, compared with least squares (LS) and MMSE algorithms, the proposed algorithm can improve the performance of delay estimation.

1 Introduction

Direct sequence spread spectrum (DSSS) is widely used in navigation, radar, and communication [1]. The delay estimation of DSSS is necessary for accurate code synchronization and applications such as range. Multipath propagation is a common error source for delay estimation of DSSS. A large number of multipath signal parameter estimation strategies have been proposed, including the extended Kalman filter (EKF), the multipath estimation delay-locked loop (DLL), and the deconvolution algorithms [2].

The EKF and MEDLL methods have a good performance on distance multipath mitigation. The EKF algorithms ([3]) construct a first-order Gauss-Markov process by taking the amplitude, carrier phase, and multipath delay as state variables. MEDLL algorithms ([4]) estimate the multipath parameters based on the maximum likelihood (ML) criterion. Each estimated parameter needs to be ergodically searched on the delay dimension. Both EKF and MEDLL algorithms have high estimation performance under the scenarios of distance multipath. But they are sensitive to the initial value and have high computational complexity. Their anti-interference ability for closely-spaced multipath is only moderate.

Deconvolution methods can deal with both distance and closely-spaced multipath interference. According to estimation criteria, they can be classified into least squares (LS) and minimum mean square error (MMSE) algorithms. The LS algorithms ([5], [6]) provide high estimation performance under a high carrier to noise ratio (CNR) with a low computational cost. However, they do not consider noise power in their cost function. The estimation performance deteriorates with the decline of CNR. MMSE algorithms ([7]-[9]) add background noise power in their cost function and have better anti-noise ability. At the receiver, the real values of multipath parameters are unknown. Therefore, the existing MMSE algorithms construct their deconvolution matrix by convex set projection (CSP) or a reiterative process. CSP-MMSE algorithms ([7]) update the deconvolution matrix by least mean square filter, which has low complexity but requires a precise design for the step parameter. The MMSE algorithm based on the reiterative process ([8], [9]) has higher estimation performance. However, the complexity is comparatively high. Moreover, both LS and MMSE algorithms construct the deconvolution matrix by a correlation function dictionary. The performance of parameter estimation increases with the similarity between correlator outputs and dictionary elements. However, in the implementation process, the density of the dictionary is limited. The difference between the dictionary and the real correlator outputs leads to an error term.

To mitigate the model error, we set a group of correction parameters in the correlation function dictionary, then, add dictionary correction in the existing MMSE algorithm. The basic of the dictionary correction is the ML estimation of correction parameters. As the correction parameters are sparse, we only estimate the parameters corresponding to high signal power and set the other parameters to zeros. This selection can avoid the unnecessary computational cost. Simulation results show that the proposed algorithm can improve the estimation performance on delay and power for the direct-path signal.

2 Signal Model

Under the assumption of perfect carrier frequency synchronization at the receiver, the baseband samples can be expressed by [4]:

$$y_n = \sum_{l=0}^{L-1} A_l b\left(nT_s - \tau_l\right) c\left(nT_s - \tau_l\right) + v_n, \tag{1}$$

where L is the number of paths, the number of direct-path is 0, A_l is the complex amplitude of lth path, $b(\cdot)$ is the symbol, $c(\cdot)$ is the DSSS sequence, τ_l is the delay of lth path, T_s is the sampling interval, and v_n is zero-mean white Gaussian noise with variance σ^2. The coarse estimation based on grid search can reduce estimation error of direct-path delay to -0.5∼1chip. Without loss of generation, we set the path with the number 0 as direct-path and the uncertainty range of τ_0 as -0.5∼1chip. At the scenarios of closely-spaced multipath, $\tau_l - \tau_0 < 0.5$chip.

Assume that the number of coherent integration points is N, corresponding to the integration time $T_{coh} = NT_s$. The input of correlator is $\mathbf{y} = \left[y_0\ y_1\ \cdots\ y_{N-1}\right]^{\mathrm{T}}$. The existing MMSE algorithm approximates $\mathbf{y}$ as a linear combination of reference signals $\mathbf{c}_m = \left[c(-\bar{\tau}_m)\ c(T_s-\bar{\tau}_m)\ \cdots\ c(NT_s-T_s-\bar{\tau}_m)\right]^{\mathrm{T}}$, $m = 0, 1, \ldots, M-1$, where $\bar{\tau}_m = \bar{\tau}_0 + m\Delta\tau$. The weight coefficient of reference signals is given by:

$$h_m = \begin{cases} A_l b_0, & \bar{\tau}_m - \frac{\Delta\tau}{2} \le \tau_l \le \bar{\tau}_m + \frac{\Delta\tau}{2} \\ 0, & others \end{cases}. \qquad (2)$$

Herein, the impact of symbol change can be ignored when the symbol duration is larger than or equal to coherent integration time (i.e. $b(nT_s - \tau_l) \equiv b_0$, $n = 0, 1, \ldots, N-1$). Thus, b_0 is used to express $b(nT_s - \tau_l)$. Further, $\mathbf{y}$ can be expressed as:

$$\mathbf{y} = \mathbf{C}^{\mathrm{H}}\mathbf{h} + \mathbf{v}, \qquad (3)$$

where $\mathbf{C} = \left[\ \mathbf{c}_0\ \mathbf{c}_1\ \cdots\ \mathbf{c}_{M-1}\ \right]$, $\mathbf{h} = \left[h_0\ h_1\ \cdots\ h_{N-1}\right]^{\mathrm{T}}$, and $\mathbf{v} = \left[v_0\ v_1\ \cdots\ v_{N-1}\right]^{\mathrm{T}}$. A vector of parallel correlator outputs with interval T_d can be expressed as:

$$\mathbf{z}_m = \bar{\mathbf{C}}_m^{\mathrm{H}}\mathbf{y} = \mathbf{D}_m^{\mathrm{H}}\mathbf{h} + \bar{\mathbf{C}}_m^{\mathrm{H}}\mathbf{v}, \qquad (4)$$

where $\mathbf{z}_m = \left[\ z_{m,-K}\ \cdots\ z_m\ \cdots\ z_{m,K}\ \right]^{\mathrm{T}}$, $\bar{\mathbf{C}}_m = \left[\ \bar{\mathbf{c}}_{m,-K}\ \cdots\ \bar{\mathbf{c}}_{m,0}\ \cdots\ \bar{\mathbf{c}}_{m,K}\ \right]$, $\bar{\mathbf{c}}_{m,k} = \left[\ c(kT_d - \bar{\tau}_m)\ c(T_s + kT_d - \bar{\tau}_m) \cdots c(NT_s - T_s + kT_d - \bar{\tau}_m)\right]^{\mathrm{T}}$, $\mathbf{D}_m = \bar{\mathbf{C}}_m\mathbf{C}^{\mathrm{H}}$. The element of $\mathbf{D}_m$ in the pth row and the qth column is $D_m(p,q) = R(\bar{\tau}_m - (p-K)T_d - \bar{\tau}_q)$, where $R(\tau)$ is the correlation function of the DSSS sequence $c(t)$ delayed by τ.

3 Proposed Algorithm

It can be seen that the existing MMSE algorithm approximates τ_l to its closest $\bar{\tau}_m$. The dictionary matrix $\mathbf{D}_m$ is mismatched to the real correlation outputs. Thus, we construct a corrected dictionary matrix $\mathbf{D}'_m$, whose elements can be expressed as

$$D'_m(p,q) = R(\bar{\tau}_m - (p-K)T_d - \bar{\tau}_q + \mu_m) \qquad (5)$$

with correction parameter

$$\mu_m = \begin{cases} \tau_l - \bar{\tau}_m, & \bar{\tau}_m - \frac{\Delta\tau}{2} \le \tau_l \le \bar{\tau}_m + \frac{\Delta\tau}{2} \\ 0, & others \end{cases}. \qquad (6)$$

Then, $\mathbf{z}_m$ can be rewritten as:

$$\mathbf{z}'_m = \mathbf{D}'^{\mathrm{H}}_m\mathbf{h} + \bar{\mathbf{C}}_m^{\mathrm{H}}\mathbf{v} = \mathbf{u}_m + \bar{\mathbf{C}}_m^{\mathrm{H}}\mathbf{v}. \qquad (7)$$

3.1 MMSE Estimation

Based on the MMSE criterion, the cost function of h_m estimation is given by:

$$J_m = \mathrm{E}\left|h_m - \hat{h}_m\right|^2 = \mathrm{E}\left|h_m - \mathbf{w}_m^{\mathrm{H}}\mathbf{z}'_m\right|^2 \qquad (8)$$

Herein, the $\hat{h}_m$ is obtained by an inverse filter with coefficient vector $\mathbf{w}_m$. To minimize the cost function, the $\mathbf{w}_m$ needs to satisfy

$$\partial J_m / \partial \mathbf{w}_m^{\mathrm{H}} = \mathrm{E}\left[\mathbf{z}'^{\mathrm{H}}_m\mathbf{z}'_m\right]\mathbf{w}_m - \mathrm{E}\left[h_m^*\mathbf{z}'_m\right] = 0. \qquad (9)$$

Then, $\mathbf{w}_m$ is given by:

$$\begin{aligned} \mathbf{w}_m &= \left(\mathrm{E}\left[\mathbf{z}'_m\mathbf{z}'^{\mathrm{H}}_m\right]\right)^{-1}\mathrm{E}\left[h_m^*\mathbf{z}'_m\right] \\ &= \left(\mathbf{u}_m\mathbf{u}_m^{\mathrm{H}} + \sigma^2\mathbf{C}_m^{\mathrm{H}}\mathbf{C}_m\right)^{-1}\mathrm{E}\left[h_m^*\mathbf{u}_m\right] \\ &= h_m^*\left(\mathbf{u}_m\mathbf{u}_m^{\mathrm{H}} + \sigma^2\bar{\mathbf{D}}\right)^{-1}\mathbf{u}_m, \end{aligned} \qquad (10)$$

where $\mathbf{u}_m = \mathbf{D}'_m\mathbf{h}$, $\mathbf{C}_m^{\mathrm{H}}\mathbf{C}_m$ is the auto-correlation matrix of local replica signal, and $\mathbf{C}_m^{\mathrm{H}}\mathbf{C}_m = \mathbf{C}_n^{\mathrm{H}}\mathbf{C}_n$ $(m \ne n)$. Hence, we define a local code dictionary matrix $\bar{\mathbf{D}}$ with element $\bar{D}(p,q) = R(pT_d - qT_d)$ to express $\mathbf{C}_m^{\mathrm{H}}\mathbf{C}_m$, $m = 0, 1, \ldots, M-1$.

As μ_m and h_m are unknown at receiver, a reiterative process is required. In each iteration, $\mathbf{u}_m$ is constructed by the estimated values $\hat{h}_m$ and corrected dictionary matrix $\mathbf{D}'_m$. The $\hat{h}_m$ is obtained by the previous iterations. At the beginning to the reiterative process, $\hat{h}_m$ is initialized to $z_{m,0}$. The detail of $\mathbf{D}'_m$ correction is provided in the following part.

3.2 Dictionary Correction

The core of the dictionary correction is the estimation of correction parameters μ_m, $m = 0, 1, \ldots, M-1$. In this work, we estimate μ_m by ML search. Let us define search range as $(-\Delta\tau/2\ ,\Delta\tau/2\ ]$, and the search points as N_s. Then, $\hat{\mu}_m$ can be expressed as:

$$\hat{\mu}_m = \underset{\mu'_{m,i}}{\arg\max}\left|\mathbf{c}'^{\mathrm{H}}_{m,i}\mathbf{y} - \sum_{n=0,n\ne m}^{M-1} R\left(\bar{\tau}_m + \mu'_{m,i} - \bar{\tau}_n\right)\hat{h}_n\right|, \qquad (11)$$

where $\mathbf{c}'_{m,i} = \left[\ c\left(-\bar{\tau}_m - \mu'_{m,i}\right)\ c\left(T_s - \bar{\tau}_m - \mu'_{m,i}\right)\ \cdots\ c\left(NT_s - T_s - \bar{\tau}_m - \mu'_{m,i}\right)\right]^{\mathrm{T}}$, and $\mu'_{m,i} = i\Delta\tau/N_s$. At the beginning to the reiterative process, $\hat{\mu}_m$ is initialized to 0.

It should be noted that, in the case of $\rho_n < \rho_m$ ($\rho_m = h_m^{\mathrm{H}}h_m$), the influence of μ_n on $\hat{h}_m$ is significantly less than the influence of μ_m on $\hat{h}_n$. In addition, when $\rho_m = 0$, it is meaningless to correct $\hat{\mu}_m$. Therefore, in the iterative process, we use threshold detection to pick out the $\hat{\mu}_m$ corresponding high ρ_m as the estimation object. The others $\hat{\mu}_m$ are set to 0. Because that $\max\rho_m$ corresponds to the power of direct-path signal, the threshold can be set by $V_h = \beta(\max\hat{\rho}_m)$, $\beta \in [0,1)$. The value of β is related to actual application requirement.

3.3 Reiterative Process

Based on the above analysis, the proposed algorithm alternately performs weight coefficient estimation and dictionary correction. The weight coefficient estimation is realized by the MMSE inverse filter. The dictionary correction is divided into three steps. First, sift the correction parameters μ_m that need to be estimated; second, update the selected μ_m by ML search and set other μ_m to 0; third, reconstruct dictionary matrix $\mathbf{D}'_m$. The algorithm structure is shown in Figure 1. The detailed reiterative process is shown in Algorithm 1. Herein, N_R is the number of iterations.

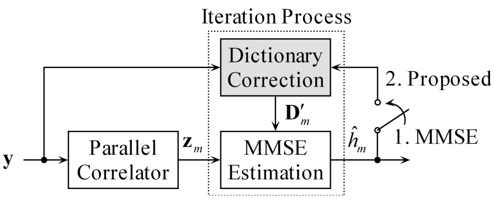


Figure 1. Block diagram of the proposed algorithm.

Algorithm 1 Proposed algorithm

Input: Received signal $\mathbf{y}$.

Compute correlator outputs $\mathbf{z}_m$, $m = 0, 1, \dots, M-1$.
Initialize $\hat{\mu}_m = 0$, $\hat{h}_m = z_{m,0}$, and threshold coefficent β.
for $i = 1 : N_R$ **do**
  Compute $\mathbf{w}_m$ by (10), $m = 0, 1, \dots, M-1$.
  Compute $\hat{h}_m = \mathbf{w}_m^{\mathrm{H}} \mathbf{z}'_m$, and update $\hat{\rho}_m$, m=0,1,. . .,M–1.
  Compute the threshold $V_h = \beta \max(\hat{\rho}_m)$.
  Select $\hat{\mu}_m$ that corresponds to $\hat{\rho}_m \geq V_h$.
  Compute the selected $\hat{\mu}_m$ by (11).
  Update dictionary matrix $\mathbf{D}'_m$ by (5), m=0,1,. . .,M–1.
end for

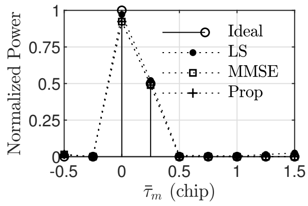
Output: $\hat{h}_m$, $m = 0, 1, \dots, M-1$.

4 Numerical Simulation

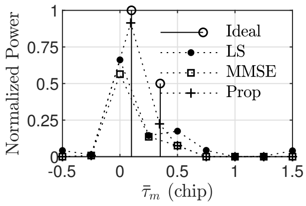
According to the conclusion in [2], [4], the performance of MEDLL algorithms is greatly affected by the initial value under the close-space multipath scenarios. Thus, we use LS and MMSE algorithms as references to verify the performance of the proposed algorithm. In this section, the number of paths is 2, the difference of delay between two paths is 0.25chip, the integration time T_{coh} is 1 ms, the number of correlators is 5, the distance between correlators T_d is 0.25chip, the uncertainty region of τ_l is -0.5∼1.5chip, the search interval $\Delta\tau$ is 0.25chip, and the number of ML search points is 8. The statistical results were obtained by 1,000 Monte Carlo simulations.

4.1 Effect of Model Error and Noise

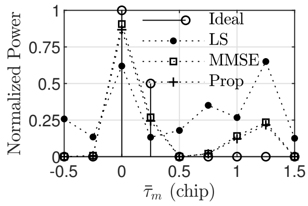
Figure 2 (a) and (b) show the normalized estimated signal power ρ_m/ρ_0 under $\tau_0 = 0$chip. In this case, there is no model error in the dictionary of LS and MMSE algorithms. When CNR=55dB, all algorithms have similar small estimation errors. With rising noise power, the estimation performance of the LS algorithm significantly decreases. Figure 2 (c) and (d) show the power estimation results under $\tau_0 = 0.1$chip. For LS and MMSE algorithms, the dictionary model is mismatched to the correlator outputs. Under high CNR, the peak of the LS and MMSE algorithms locate at 0.1chip away from the true delay, and the estimated power of the direct-path signal is obviously lower than its true value. When CNR=45dB, only the proposed algorithm shows the correct peak location. Comparatively, the proposed algorithm provides better estimation performance by mitigating the model error.



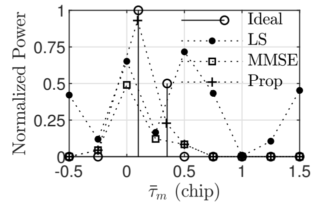
(a) CNR=55dBHz, $\tau_0 = 0$chip.



(b) CNR=45dBHz, $\tau_0 = 0$chip.



(c) CNR=55dBHz, $\tau_0 = 0.1$chip.



(d) CNR=45dBHz, $\tau_0.1 = 0$chip.

Figure 2. Normalized estimated signal power.

4.2 Estimation performance

Figure 3 shows the root mean square error (RMSE) of the direct-path delay under CNR=45dBHz. The x-axis is set in the range of $[0, \Delta\tau/2\,]$. The RMSE of LS and MMSE algorithms monotonically increase with τ_0. For the LS algorithm, the RMSE rises from 0.4chip to 0.6chip. For the MMSE algorithm, the RMSE rises from 0.25chip to 0.4chip. The RMSE of the proposed algorithm fluctuates around 0.25 chip. In most cases, the proposed algorithm has the best estimation performance. Figure 4 shows the normalized mean square error (NMSE) of weight coefficient (i.e. $\sum |h_m - h_m|^2/\rho_0$) under CNR=45dBHz. The curves of all algorithms monotonically increase. Comparatively, the proposed algorithm has the lowest error.

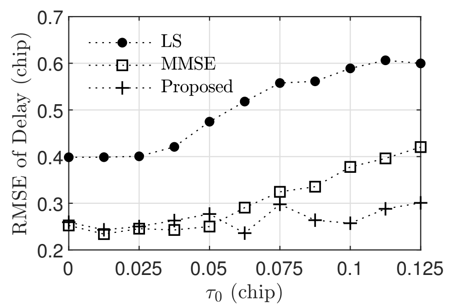


Figure 3. Estimation error of delay under different τ_0.

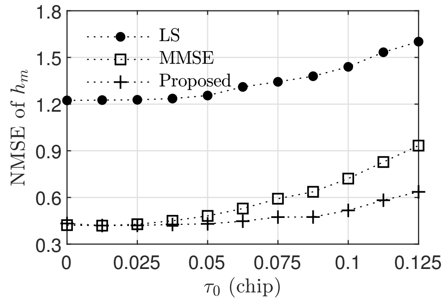


Figure 4. NMSE of $\hat{h}_m$ under different τ_0.

Figure 5 and Figure 6 show the RMSE of delay and NMSE of weight coefficient under different CNR, respectively. τ_0 uniformly distributes in the range of $[0, \Delta\tau/2\,]$. As shown, the MMSE and proposed algorithms have better performance under low CNR. With the rising CNR, the gap between the LS and MMSE algorithms decreases. The difference between the MMSE and the proposed algorithms is more obvious under high CNR. It is caused by the sensitivity of ML search to noise.

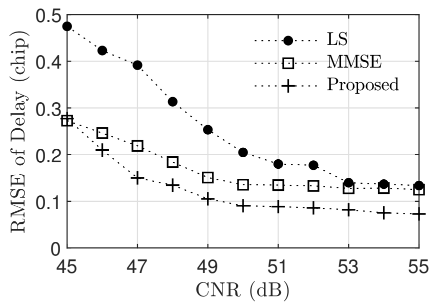


Figure 5. Estimation error of delay under different CNR.

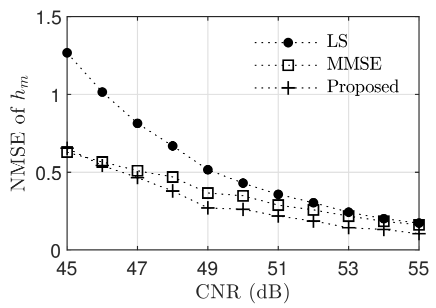


Figure 6. NMSE of $\hat{h}_m$ under different CNR.

5 Conclusion

In this work, we set a group of correction parameters in the dictionary matrix to provide an accurate model for the MMSE inverse filter. The dictionary matrix is reconstructed based on the estimated correction parameters. The estimation error of the direct-path signal power can be reduced with the refined dictionary matrix. Simulation results show that, in the scenarios of closely-spaced multipath, the proposed algorithm can improve estimation performance on delay and power of direct-path signal.

References

[1] R. Morales-Ferre, P. Richter, E. Falletti, A. Fuente, and E. Lohan, “A Survey on Coping With Intentional Interference in Satellite Navigation for Manned and Unmanned Aircraft,” *IEEE Communications Surveys & Tutorials*, **22**, 1, January 2020, pp. 249–291, doi:10.1109/COMST.2019.2949178.

[2] E. S. Lohan, R. Hamila, and A. Lakhzouri, “Highly efficient techniques for mitigating the effects of multipath propagation in DS-CDMA delay estimation,” *IEEE Transactions on Communications*, **4**, 1, January 2005, pp. 149–162, doi:10.1109/TWC.2004.840231.

[3] M. Hernandez, and A. Farina, “PCRB and IMM for Target Tracking in the Presence of Specular Multipath,” *IEEE Transactions on Aerospace and Electronic Systems*, **56**, 3, June 2020, pp. 2437–2449, doi:10.1109/TAES.2019.2948671.

[4] P. Wang, and Y. J. Morton, “Improved Time-of-Arrival Estimation Algorithm for Cellular Signals in Multipath Fading Channels,” in IEEE/ION Position, Location and Navigation Symposium (PLANS), Portland, Oregon, USA, 2020, doi: 10.1109/PLANS46316.2020.9110178.

[5] E. Schmidt, and N. Gatsis, and D. Akopian, “A GPS Spoofing Detection and Classification Correlator-Based Technique Using the LASSO,”*IEEE Transactions on Aerospace and Electronic Systems*, **56**, 6, December 2020, pp. 4224–4237, doi:10.1109/TAES.2020.2990149.

[6] G. Zhou, and L. Wang, “Pseudo-Spectrum based speed square filter for track-before-detect in range-doppler Domain,”*IEEE Transactions on Aerospace and Electronic Systems*, **67**, 21, November 2019, pp. 5596–5610, doi:10.1109/TSP.2019.2943256.

[7] Y. Cai, X. Wu, M. Zhao, and R. Lamare, “Low-complexity reduced-dimension space-time adaptive processing for navigation receivers,”*IEEE Transactions on Aerospace and Electronic Systems*, **54**, 6, December 2018, pp. 3160–3168, doi:10.1109/TAES.2018.2829380.

[8] T. Cuprak, and K. Wang, “Efficient Doppler-compensated reiterative minimum mean-squared-error processing,”*IEEE Transactions on Aerospace and Electronic Systems*, **53**, 2, April 2017, pp. 562–574, doi:10.1109/TAES.2017.2651480.

[9] Y. Shen, and Y. Xu, “Multiple-access interference and multipath influence mitigation for multicarrier code-division multiple-access signals,”*IEEE Access*, **8**, January 2020, pp. 3408–3415, doi:10.1109/ACCESS.2019.2962633.