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On bandwidth bounds for periodic antenna elements with a ground plane

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Abstract

In this paper, we compare two different bandwidth bounds on array antennas with a ground plane. The first of these bounds is based on the fact that the antenna can be seen as a passive system, which implies that its response can be described in terms of a Herglotz function. This passivity-based bandwidth bound is closely related to a sum rule for lossless arrays over the ground plane. The second bound is a recently developed Q-factor based bound, valid for resonant structures. In this paper, it is shown that this latter bound strongly depends on the allowed shape and the electrical size of the element within the unit cell. It also depends on the size of the unit cell, corresponding to the inter-element spacing. The Q-factor bound provides a sharper bound when the structure is resonant.

1 Introduction

An essential design parameter for electromagnetic devices in general and antennas, in particular, is their operating bandwidth. The bandwidth is in a trade-off relation with other antenna design parameters. For example, increasing the gain of an antenna tends to reduce the available bandwidth [1, 2]. Similarly, reducing the electrical size of the antenna reduces the available bandwidth [3]. For electrically large and wideband structures, such as array antennas or absorbers, the array figure of merit has been developed to show some of the trade-offs that impact the bandwidth. An essential parameter in this trade-off is how the thickness of the antenna above its ground plane impacts the best possible bandwidth, see [4, 5, 6].

There is, however, a category of resonant periodic structures above a ground plane, that are both electrically large and for which the array-figure of merit tend to overestimate the available bandwidth. For these structures, the periodic Q-factor bound can be used to predict the available bandwidth. In our recent paper [7], a Q-factor representation in terms of the surface currents has been developed and validated. The expression applies to resonant structures, e.g., with sufficiently high Q, in a rectangular unit cell in 2D-periodic lossless structures above a ground plane. Optimization over all possible currents in a region provides the Q-factor bound for the given region.

It is interesting to compare these two tools, that are based on different assumptions. The assumptions for the array figure of merit-bound are linearity, passivity, time-invariance, and that the structure is lossless. It is based on a sum rule for the reflection coefficient. The low-frequency asymptotic depends on the electric and magnetic polarizabilities. The appearing linear combination is estimated from above with the thickness of the structure [8]. For broadside radiation, the array figure of merit and consequently the antenna fractional bandwidth can thus be expressed in terms of the allowed reflection coefficient, the thickness, and the maximal static material parameters.

The Q-factor bound is based on stored energies and accurately predicts bandwidth of resonant structures. Here we consider perfectly conducting structures (PEC). The Q-factor accounts for the allowed geometry, electrical size, and position above the ground plane of the structures within the unit cell. Trade-offs between differently sized and shaped antenna elements and their relative position for different periodicities are addressed in this paper.

2 Theory

In the introduction, two different approaches to bandwidth bounds were discussed: The array figure of merit [5] and the Q-factor based bandwidth bound. The array figure of merit, in its simplest form, is reformulated here as an upper bound on the available fractional bandwidth, B:

$$B \leq \frac{2\alpha(\theta)h/\lambda_0}{1+\sqrt{1+(\alpha(\theta)h/\lambda_0)^2}}. \tag{1}$$

Here, h is the thickness of the structure above the ground plane, λ_0 is the wavelength at the center frequency, θ is the scan direction as a polar angle away from the broadside direction. The constant α for broadside radiation is

$$\alpha(0) = \frac{40\pi^2\mu_s\log(\mathrm{e})}{RL} \approx \frac{171.5\mu_s}{RL}, \tag{2}$$

where RL is the return loss in dB and μ_s is the maximal static relative permeability in the domain. For the cases discussed here, let the static permeability be $\mu_s = 1$ and the return loss is 10 dB, resulting in that $\alpha(0) \approx 17.1$. The underlying assumption of this result is the passivity of the structure, linearity, time-translational invariance, and that the structure is lossless. As losses tend to increase the bandwidth, the obtained bandwidth is smaller in the lossless case. It is valid for arrays with a ground plane and it is an upper bound on the available fractional bandwidth. In this simple form, the essential parameters are the chosen level of return loss and the wavelength-normalized thickness of the structure above the ground plane.

To design structures that reach the sum-rule based bound (1) is challenging. For certain well-designed wideband structures it appears to be tight, while most array element designs are far from the bound [5]. It thus appears to overestimate the available bandwidth in particular for resonant structures. In this situation, it is interesting to use a Q-factor bound as it gives a tighter bound on the bandwidth.

The Q-factor for periodic structures was developed in [7, 9], extending the earlier results of [10, 11, 12]. The idea is based on electric and magnetic stored energies. The electric stored energy is defined by considering the integral of $|\vec{E}|^2 - |\vec{E}_{\mathrm{P}}|^2$ over a column based in the unit-cell, $\mathbb{R}_+ \times C$, where $C = [0, p_{\mathrm{x}}] \times [0, p_{\mathrm{y}}] \subset \mathbb{R}^2$ is the unit cell, and $\mathbb{R}_+ = \{x \in \mathbb{R} : x \geq 0\}$. Above, $\vec{E}$ is the electric field, and $\vec{E}_{\mathrm{P}}$ is the propagating modes of the field. To formulate the electric and the corresponding magnetic stored energies in terms of the volume-current densities, $\vec{J}$, the Green's-function approach is used. This results in quadratic forms for the stored electric energy, W_{e}, the stored magnetic energy, W_{m}, and the radiated power, P_{rad}:

$$W_{\mathrm{e}} = \langle \vec{J}, K_{\mathrm{e}} \vec{J} \rangle,\ W_{\mathrm{m}} = \langle \vec{J}, K_{\mathrm{m}} \vec{J} \rangle,\ P_{\mathrm{rad}} = \langle \vec{J}, K_{\mathrm{r}} \vec{J} \rangle. \quad (3)$$

Here, $\langle a, b \rangle = \int a^* b \, \mathrm{d}V$, and the operators K_{e}, K_{m} and K_{r} are explicitly given in [7]. These operators are similar in complexity to compute as the components of the electric-field integral-equation operator [13]. The result depends on the shape and position of the antenna element.

The tuned Q-factor is defined as

$$Q = \frac{2\omega \max(W_{\mathrm{e}}, W_{\mathrm{m}})}{P_{\mathrm{diss}}}, \quad (4)$$

where ω is the angular frequency and P_{diss} is the dissipated power. In the lossless case, $P_{\mathrm{diss}} = P_{\mathrm{rad}}$. The current-density optimization approach to the Q-factor bound is to define an antenna region within the unit-cell, and to minimize Q over all possible currents in the region.

The relation between Q and the fractional bandwidth for resonant isolated antennas is well known [14]:

$$B \approx \frac{2|\Gamma|}{Q\sqrt{1 - |\Gamma|^2}}, \quad (5)$$

where $|\Gamma|$ is the lowest desired reflection coefficient. The relation also remains valid for resonant large periodic arrays, see [7, 9].

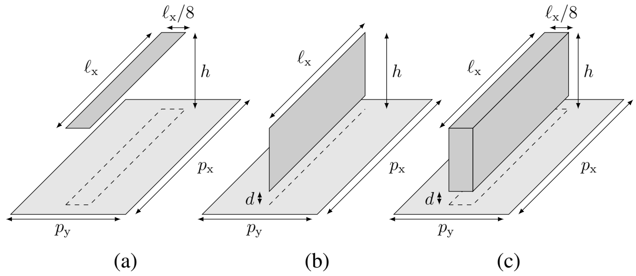

Figure 1. Geometries considered for the Q-factor optimization.

3 Numerical Results

While the array figure of merit is simple enough to be an analytical formula in terms of the wavelength normalized height and the return loss (1), the Q-factor approach requires numerical calculations. For perfectly electric conducting structures, volume-current densities reduce to surface-current densities and the volume integral in the $\langle \cdot, \cdot \rangle$ reduces to a surface integral over the element shape. A triangular mesh is used to mesh the antenna elements. The 2D-periodic Green's function in a 3D-setting is implemented with Ewald's method. The stored energies and the radiated power are approximated as quadratic forms using the Method-of-Moments approach with Rao-Wilton-Gilsson basis functions. The procedure is similar to the computation of the impedance matrix that corresponds to the electric field integral-equation operator. The optimal Q-factor is reduced to a Quadratically Constrained Quadratic Program:

$$\begin{aligned} Q_* = &\min\ 2\omega \max(W_{\mathrm{e}}, W_{\mathrm{m}}), \\ &\text{subject to } P_{\mathrm{rad}} = 1. \end{aligned} \quad (6)$$

This optimization is performed over all possible surface current densities on the enclosure of the allowed antenna element region. The idea behind the Q-factor bound is that among all possible currents in a region, at least one current density represents the best realizable antenna within the given region [3].

To compare the Q-factor bound with the passivity-based bound, consider the three shapes shown in Fig. 1. The solution to the optimization problem as a function of the wavelength-normalized height is depicted in Fig. 2. The upper bound on the available bandwidth for any PEC shape contained within a given wavelength-normalized thickness, h/λ_0 is depicted with the upper (black) curve from the passivity based sum-rule bound. It is the upper bound to all considered shapes and heights.

The blue, red and green solid lines in Fig. 2 are a cuboid of relative dimensions $\ell_x \times \frac{\ell_x}{8} \times (h - d)$, for three cases of unit-cell widths $p_y = (1, \frac{1}{2}, \frac{1}{4}) p_x$. The relative length of the unit cell is $p_x = \lambda_0/2$, with $\ell_x/p_x = 0.9$ and $d/p_x = 0.01$. The two planar cases: horizontal (dashed) and vertical (dotted)

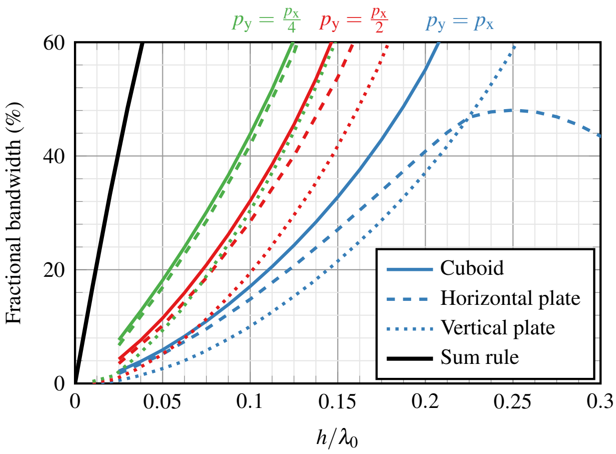

Figure 2. Bounds on the bandwidth. The top curve is the sum-rule bound. The remaining curves are Q-factor based trade-off curves for different shapes, with reflection coefficient that is smaller than -10 dB.

are positioned at the center of the unit cell and of the respective relative size $\ell_x \times \ell_x/8$ and $\ell_x \times (h-d)$ and the plates are negligibly thin. The cuboid is the uppermost curve, for all cases of different unit-cell width. For small values of h/λ_0, note that the vertical plate provides the smallest bandwidth. When the height approaches $\lambda_0/4$, the bandwidth of the vertical plate exceeds that of the horizontal plate. The cause behind the better performance of the horizontal plate for sufficiently thin structures is that on the vertical plate, most current tries to accumulate at the top, where there is very little space. As a comparison, for the horizontal plate, all currents are as high as possible above the ground plate. It is thus advantageous to use a horizontal plate for arrays that are thin.

In [7], an examination was made on how well the Q-factor predicts the bandwidth for a series of array antennas and over a range of frequencies. There it was shown that $Q \sim 5$ and larger tend to well predict the fractional bandwidth. We note here that a 20% fractional bandwidth corresponds to $Q \sim 3.3$ at 10 dB return loss. Thus, in the higher regions of thickness and/or more narrow unit-cells it is clear that the structure might have multiple resonances, and the passivity based sum-rule should be used as the upper bound. The increase in the Q-factor-based bandwidth bound as the p_y is reduced is clear, since the density of regions where the current can be localized is increasing.

4 Conclusion

In this paper, we have compared the array-figure-of-merit bandwidth bound with the Q-factor based bandwidth bound. For the Q-factor, we discussed the bounds on the planar plate, the horizontal plate, and the cuboid. They provide an upper bound on any resonant antenna element shape that fits within the respective region region. Furthermore, we note that the unit-cell periodicity strongly impacts the upper bound of the available bandwidth for these Q-factor based bounds. The Q-factor bound is based on the assumption that a Q-factor can be used to approximate the array's resonance. Such approximation accurately represents bandwidth for resonant structures. Previous studies have shown that $Q \sim 5$ or larger tend to be resonant. For wideband structures we use the array-figure-of-merit bound that is an upper bound for the resonant structures. We note here, that in the region of exceptionally small Q, we find that the two bounds may cross. In this region it is clear that the array-figure-of-merit bound then provides a tighter bound on the structure. The Q-factor bound in the resonant region provides a method for direct testing of what shape that has the better, larger bandwidth, behavior. That can provide direct input to choices of design questions.

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