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Evaluation of MoM Reaction Integrals Applying the Divergence Theorem

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Filling the system matrix is a key step in the numerical solution of integral equations discretized via the method of moments (MoM). The reaction integrals that constitute the system matrix elements need to be evaluated accurately, especially for matrix elements representing near-field interactions, where the integrals are singular or near-singular. At the same time, a fast evaluation procedure is desired. Moreover, the implemented integration scheme is expected to be general purpose and robust with respect to discretization cell shapes and aspect ratios. The published literature on MoM reaction integrals generally focuses on the accurate evaluation of the source (inner) integral, which may be singular or near-singular. A traditional approach employs the so-called "singularity subtraction" scheme, where an asymptotic form of the (near-)singular behavior of the integrand is derived which is subtracted from the original integrand to yield a smoothed integrand for integration via standard quadrature rules [1]; the subtracted asymptotic form is then additively restored as an *analytically* evaluated integral. In so-called "singularity cancellation" schemes, on the other hand, proper variable transformations are applied to the source integral in order to cancel its (near-)singularities; it is then evaluated numerically [2][3]. However, in both approaches, which are often able to reach machine precision, all the effort is in the accurate evaluation of the source integral with little attention to the test (outer) integral; near source domain boundaries, however, the latter may also contain bounded or unbounded singularities, rendering standard quadrature schemes inefficient [4]. In this contribution, we explore a different framework, wherein the reaction integral is treated in its entirely. Applying the divergence theorem separately to the both source and test integrals, the original double surface/volume reaction integral is rewritten as two radial integrals plus two (outer) contour/surface integrals. The resulting radial integrals are found to be well-behaved, and the accuracy of the contour/surface integrals can be easily improved via proper variable transforms, often achieving machine precision accuracy in the evaluation of the entire reaction integral. In this novel formulation, the integrals can be evaluated with fully numerical schemes or hybrid analytically/numerical techniques. In this work, we will first review the proposed approach for double surface integrals [5][6], then present recent results in the application of the divergence theorem to double volume integrals.

References

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