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A Generalized Source Integral Equations Fast Direct Solver

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Low-rank compression-based integral equation solvers have become commonly used for treatment of intermediate electrical size ill-conditioned problems, when many excitations are of interest. A direct solver computes, once, a compressed form of the impedance matrix's inverse, which can then be applied to multiple right-hand-sides, and does not rely on iterative convergence. The low-rank compression reduces the asymptotic computational complexity of the algebraic direct solution process, but only for geometries and integral equation kernels that give rise to inherently rank deficient impedance matrix blocks. Such are the cases of elongated and quasi-planar wave problems, where the rank increases, with the problem's dimensions, slower than the number of problem unknowns [1]. With the aim of extending the range of problems that can be efficiently solved via low-rank compression, the generalized equivalence integral equation [2] was developed. By inserting a simple scatterer into the volume occupied by closed surface, without modifying the outside solution, this equation reduces the problem's effective dimensionality and, thus, increases its low-rank compressibility. Yet, heavily relying on the internal scatterer for eliminating line-of-sight interactions between points on the original scatterer, this approach is effective mostly for objects of particular curvatures (e.g., essentially circular shapes). As an alternative, the generalized source integral equations (GSIE) were proposed [3]. In these equations, each elemental source is assigned a "shield" of auxiliary sources, which approximately cancel its radiation into the scatterer's volume and, thus, opposite side surface. The resulting integral equation blocks become more rank-deficient, for a wider range of essentially convex geometries.

This paper presents a GSIE-based fast direct solver, for scattering by essentially-convex impenetrable objects. The solver makes use of a GSIE formulation with absorbing arc shields, in conjunction with a hierarchical algebraic compression procedure [4]. We will present the considerations in the design of these sources for maximizing the rank deficiency of the impedance matrix blocks, given the desired error levels. The removal of the computational bottleneck associated with the computation of the GSIE's modified Green's function, via efficient sampling and tabulation of its components, will also be discussed. We will provide detail on the tailoring of the algebraic procedure to our choice of the GSIE shield, implemented by using non-uniform grid (NG)-based techniques, and on the resulting computational complexity. We will numerically study the compressibility obtained for various choices of shield parameters. Once the shield parameters are set, we will showcase the favorable scaling of computation time and memory requirements with the scatterer size, for representative two-dimensional examples of essentially-convex scatterers.

References

[1] E. Corona, P. G. Martinsson, and D. Zorin, "An O(N) direct solver for integral equations on the plane," *Appl. Comput. Harmon. Anal*., **38**, 2, March 2015, pp. 284-317, doi: 10.1016/j.acha.2014.04.002.

[2] A. Boag and V. Lomakin, "Generalized equivalence integral equations," *IEEE Antennas Wirel. Propag. Lett*., **11**, 2012, pp. 1568-1571, doi: 10.1109/LAWP.2012.2236294.

[3] A. Sharshevsky, Y. Brick, and A. Boag, "Conditioning of generalized source integral equation formulations," *ICEAA '19*, Granada, Spain, Sept. 2019.

[4] Y. Brick, V. Lomakin, and A. Boag, "Fast direct solver for essentially convex scatterers using multilevel non-uniform grids," *IEEE Trans. Antennas Propag*., **62**, 8, August 2014, pp. 4314-4324, doi: 10.1109/TAP.2014.2327651.