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Butterfly-based Hierarchically Semi-separable Matrix for Integral Equation Solvers

Yang Liu*(1), Pieter Ghysels(1), and Xiaoye Sherry Li(1)
(1) Lawrence Berkeley National Laboratory, Berkeley, CA, USA, e-mail: liuyangzhuan@lbl.gov; pghysels@lbl.gov; xsli@lbl.gov

Extended Abstract
Butterfly-based direct integral equation (IE) solvers are attractive alternatives to fast multipole method-based iterative solvers for solving high-frequency electromagnetic scattering and radiation problems, especially for ill-conditioned systems or problems involving multiple right-hand sides. These techniques build upon the butterfly representation of blocks in the forward and inverse IE operators, which are oftentimes less compressible using low-rank-based techniques. Previously, butterfly extensions of the $\mathcal{H}$-matrix and hierarchically off-diagonal low-rank (HOD-LR) matrix formats have been developed, which are named butterfly-LU [1] and hierarchically off-diagonal butterfly [2] (HOD-BF) formats, respectively. These formats represent off-diagonal blocks as butterfly decompositions and leverage complicated hierarchical butterfly arithmetic to achieve $O(N\log^2 N)$ compression and $O(N^{1.5}\log N)$ factorization complexities.

Here we further simplify the hierarchical representation via essentially representing the entire matrix as one butterfly decomposition plus manageable number of smaller blocks. This representation can be treated as butterfly generalization of hierarchically semi-separable (HSS) matrix format, dubbed HSS-BF. As opposed to butterfly-LU and HOD-BF that require $O(\log N)$ factors, HSS requires only $O(\log(\log N))$ factors (see Figure 1 for an example). As a result, the compression complexity can be further reduced to $O(N\log N)$. The essential operation for its factorization is the butterfly extension of the Sherman–Morrison–Woodbury inversion formula, which can be computed recursively leveraging the randomized butterfly arithmetic. In addition, a distributed-memory parallelization of the proposed format has been developed. We will compare the accuracy, efficiency and stability of the butterfly-LU, HOD-BF and HSS-BF formats. All the codes have been integrated as the software package ButterflyPACK freely available at https://github.com/liuyangzhuan/ButterflyPACK.

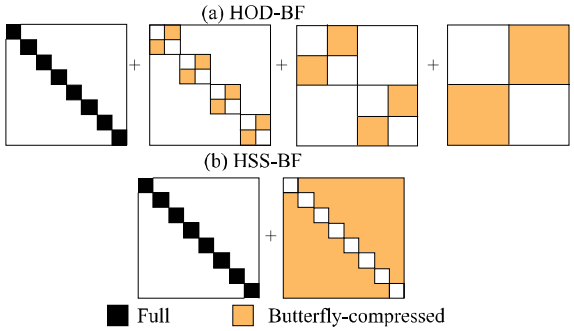

Figure 1. Illustration of (a) the HOD-BF format with $\boldsymbol{O(\log N)}$ factors and (b) the HSS-BF format with $\boldsymbol{O(\log(\log N))}$ factors.

References

[1] H. Guo, Y. Liu, J. Hu, and E. Michielssen, "A buttery-based direct integral-equation solver using hierarchical LU factorization for analyzing scattering from electrically large conducting objects," *IEEE Transactions on Antennas and Propagation*, **65**, 2017, pp. 4742-4750.

[2] Y. Liu, H. Guo, and E. Michielssen, "An HSS matrix-inspired butter y-based direct solver for analyzing scattering from two-dimensional objects," *IEEE Antennas and Wireless Propagation Letters*, **16**, 2017, pp. 1179-1183.