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Circuit virtual perfect matching enabled by complex frequency excitation

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Abstract

In this contribution, we exploit virtual perfect matching to achieve impedance matching of purely reactive loads in lossless transmission lines, by using complex frequency excitation, *i.e.* a signal having a specific temporal shape. The proposed system is lossless, since the load is purely reactive, energy is not dissipated, but stored in the load as long as the complex frequency excitation condition is satisfied, giving rise to an interesting singularity in the transient regime, also known as virtual absorption. Then, the accumulated energy is released by the load once the applied illumination changes.

1 Introduction

Perfect impedance matching in microwave transmission lines is a suitable condition since it allows the maximum power delivered to the load. If the load is resistive or complex, different impedance matching networks can be applied between the load and the considered resistive transmission line [1]. Conversely, if the load is composed of purely reactive passive lumped elements, impedance matching cannot take place, since the energy delivered to the load will be always reflected back to the source over the whole spectrum, unless a lossy matching network is designed to introduce a damping factor for the incident signal.

Recently in [2,3], we investigated on the possibility to achieve virtual matching for purely reactive passive loads terminating a lossless transmission line, by leveraging the virtual perfect absorption principle. Among several scattering anomalous phenomena [4–7], virtual absorption has been introduced to indicate the anomalous scattering behavior occurring in lossless dielectric slabs [8,9] and metasurface-bounded open cavities [10–12], when the complex zeros of scattering eigenmodes are excited. Energy is neither reflected nor absorbed, but indefinitely stored within the lossless system, and available for being released when the virtual absorption conditions are not satisfied anymore. In this contribution we present the virtual absorption concept applied to the case of lossless circuits, and identify the microwave circuit topologies and element values that makes this phenomenon possible.

Hereafter, a lossless transmission line terminated in a load composed of reactive loads in stand-alone, series or parallel connections is considered, and we demonstrate that the system exhibits virtual perfect matching if the illuminating signal is properly shaped.

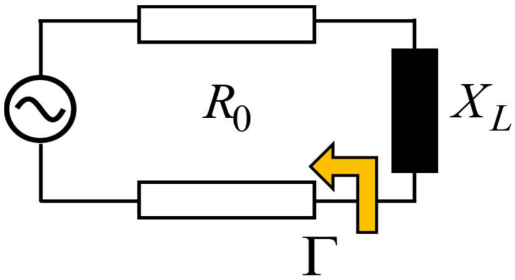

Figure 1. Transmission lines considered for virtual perfect matching. For single reactive case, $X_L = \omega L$ or $X_L = 1/\omega C$. For series connection load, $X_L = \omega L + 1/\omega C$ and for parallel connection load, $X_L = \omega L \parallel 1/\omega C$.

2 Virtual perfect matching

Let us start our analysis by considering a lossless transmission line (TL) as in Figure 1, loaded by an inductor L or a capacitor C. The reflection coefficient at the load terminals is:

$$\Gamma^L(\omega) = \frac{j\omega\tau - 1}{j\omega\tau + 1},\ \Gamma^C(\omega) = -\Gamma^L(\omega) \tag{1}$$

where the superscripts "L" and "C" identify the cases of an inductive load and a capacitive load, respectively. The quantity τ is the well-known circuit time constants for a transient step response for RL and RC circuits, and it assumes the values $\tau = L/R_0 = \tau^L$ in the inductive case, and $\tau = R_0 C = \tau^C$ in the capacitive one, where R_0 is the characteristic impedance of the feeding transmission line. This value determines how fast the time response goes to zero.

Let us now considering the same TL in Figure 1 loaded with a series/parallel connection of an inductor and capacitor. The corresponding reflection coefficients are:

$$\begin{aligned}\Gamma^{-}(\omega,\tau^C) &= \frac{(\omega^2 - \omega_{res}^2) + j(\tau^C \omega_{res}^2)\omega}{(\omega^2 - \omega_{res}^2) - j(\tau^C \omega_{res}^2)\omega}, \\ \Gamma^{//}(\omega,\tau^L) &= -\Gamma^{-}(\omega,\tau^C)\end{aligned} \tag{2}$$

where the superscripts "-" and "//" identify the series and parallel connection, respectively, and $\omega_{res} = 1/\sqrt{LC}$ is the natural resonant frequency of the LC circuit for both configurations.

It is clear that the reflection coefficients in (1) and (2) are always unitary due to the mismatch with the resistive characteristic impedance R_0 of the TL. To match the reactive loads to the TL, a method has been investigated in [13], where the reactive load has been assumed to be time-varying, in order to show a resistive behavior. However, such an approach requires fast modulation and tunable reactive loads. On the contrary, exploiting the temporal modulation of the incident signals, the reflection coefficient in (1)-(2) can vanish, leading to a perfect virtual matching at the load terminals. Temporal modulation of the signals is here introduced by considering a complex value of the excitation frequency ω, *i.e.* $\omega = \omega_r + j\omega_i$ [8,10], that corresponds to a harmonic signal of frequency ω_r, whose the amplitude is modulated over time as defined by ω_i.

Under such an excitation, eq. (1) vanish at the complex frequency $\omega_0^{L,C}$:

$$\omega_0^{L,C} = \omega_r + j\omega_i = 0 - j\tau^{-1}, \text{ with } \tau = \tau^{L,C}, \quad (3)$$

that is located along the imaginary frequency axis in the complex frequency plane shown in Figure 2 at position $\omega_x = \omega_0^{L,C}$.

In case of a load as a series/parallel connection of an inductor and a capacitor, the reflection coefficient in (2) vanishes for the complex frequency $\omega_0^{-,//}$:

$$\omega_0^{-,//} = -\frac{j}{2}\omega_{res}\left(\tau\omega_{res} \pm \sqrt{\tau^2\omega_{res}^2 - 4}\right), \text{with } \tau = \tau^{C,L} \quad (4)$$

As shown in Figure 2, such a complex frequency may assume different configurations according to the square root argument in equation (4) that may be positive, negative or identically zero, leading to a different kind of excitation:

- Positive ($\tau\omega_{res} > 2$): there are two purely imaginary zero distributed on the imaginary frequency axis (green squares in Figure 2);
- Zero ($\tau\omega_{res} = 2$): the two zeros in previous case degenerate into the imaginary counterpart of the natural resonant frequency at $\omega_x = -\omega_{res}$ (blue circle in Figure 2);
- Negative ($\tau\omega_{res} < 2$): there are two zeros having complex frequencies with a non-zero real part, $\pm j\omega_{res}$ (red crosses in Figure 2).

However, it is worth highlighting that the left singularities lay in the negative real frequencies half-plane, where $\omega_r < 0$, that cannot be excited.

3 Design of temporally modulated signals

In this section, we design the temporally modulated signals that allow vanishing the reflection coefficient at the terminals of a purely reactive load. To reach this goal, we first consider a single capacitor of $C = 5pF$ terminating a 50-Ohm transmission line. The circuit supports virtual perfect matching under the growing

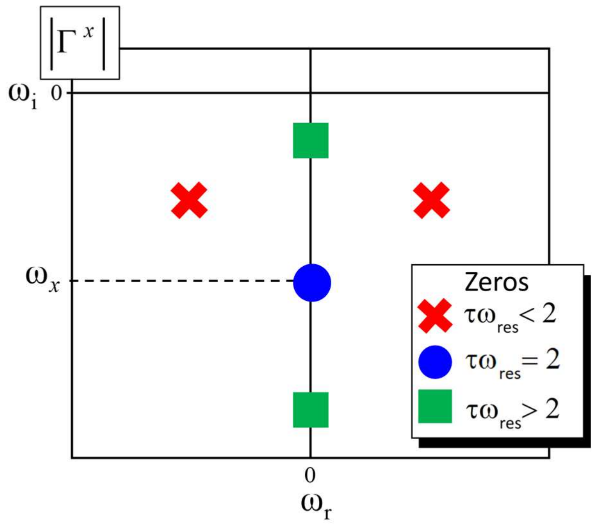

Figure 2. Reflection coefficient zeros positions in complex scattering plane for a single reactive load, thus $|\Gamma^x| = |\Gamma^{L,C}|$, showing a single zero $\omega_x = \omega_0^{L,C}$ in the lower half plane, and for LC series/parallel connections, thus $|\Gamma^x| = |\Gamma^{-,//}|$, showing: two pairs purely imaginary when $\tau\omega_{res} > 2$ (green squares), coincident purely imaginary at $\omega_x = -\omega_{res}$ when $\tau\omega_{res} = 2$ (blue circles), complex-valued when $\tau\omega_{res} < 2$ (red crosses).

envelope excitation of $\omega_i = -4 \times 10^9$, according to (3). The response in terms of reflected signal at the load terminal is reported in Fig. 3a (red curves). Remarkably, there is no reflection as long as the time-modulated signal is present at the load terminals. As soon as the signal stops, at the kick-off time t_0, the zero-reflection condition is no longer satisfied, so the circuit releases the stored energy. This anomalous behavior is showed also for LC series case satisfying $\tau\omega_{res} > 2$, where the 50-Ohm transmission line terminates in a reactive load composed of $L = 2nH$ and $C = 10pF$. The temporally modulated signal has been designed by using (4): the signal is again of purely imaginary frequency being $\tau\omega_{res} > 2$, and equals to $\omega_i = -2.2 \times 10^9$. In Figure 3(a), signals are represented with green lines and shows virtual perfect matching.

In Figure 3(b), the response in terms of reflected signal at the load terminal is reported for the case of a parallel LC circuit composed of an inductance $L = 1nH$ and a capacitor of value $C = 20pF$. The considered circuit satisfies $\tau\omega_{res} < 2$, requiring a complex frequency signal for vanishing the reflection coefficient. Using (4) we designed the temporally modulated signal, that is a monochromatic signal at $f_0 = 1.12\text{GHz}$ with a growing envelop described by $\omega_i = -5 \times 10^8$. As the signal starts, energy is provided to the circuit with the exponentially growing signal oscillating at f_0, corresponding to the green dashed signal.

4 Conclusions

Virtual perfect matching of a purely reactive loaded lossless transmission line has been investigated and achieved by exploiting the concept of virtual absorption. Energy is neither reflected nor dissipated, but just accumulated within the lossless system, as long as the proper excitation signal is applied. Then, energy becomes available for being released as soon as the virtual perfect matching condition is not satisfied anymore, by stopping or changing the tailored excitation.

5 References

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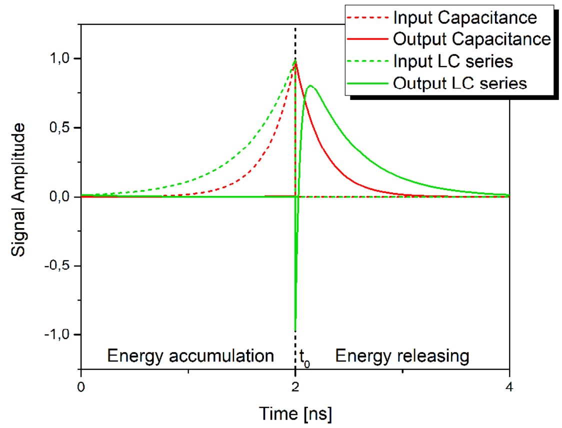
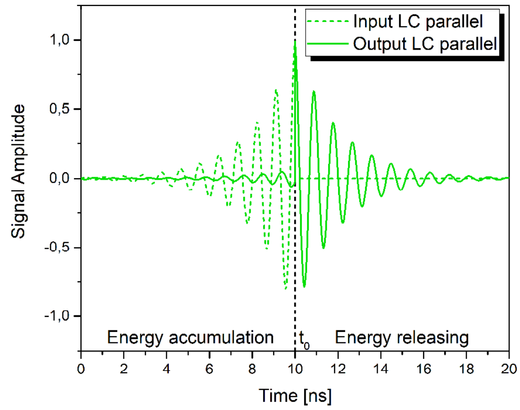


(a)



(b)

Figure 3. Transient response for (a) a single capacitor of $5\,pF$ (red lines) and LC series load with $L = 2nH$ and $C = 10\,pF$ satisfying $\tau\omega_{res} > 2$ (green lines); (b) LC parallel load with $L = 1nH$ and $C = 20\,pF$ satisfying $\tau\omega_{res} < 2$ (green lines). Incident (dashed line) and released (solid line) amplitude signals.

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