URSI GASS 2021, Rome, Italy, 28 August - 4 September 2021

Modeling and Harnessing Wave Propagation in Nonlocal and Non-Hermitian Media via Extended Transformation Optics

Massimo Moccia, Giuseppe Castaldi, Vincenzo Galdi*
Fields & Waves Lab, Department of Engineering, University of Sannio, I-82100 Benevento, Italy

Abstract

Nonlocal and non-Hermitian effects are becoming increasingly relevant in the modeling and engineering of metamaterials. Here, we review some strategies for extending the transformation-optics framework so as to enable the analysis and synthesis of these effects, while retaining the powerful and insight-providing geometrical interpretation that characterizes the conventional formulation.

1 Introduction

The study of wave propagation in complex media has been a problem of longstanding interest in electromagnetics, mostly from the academic viewpoint. More recently, there has been a revamped interest also from the application standpoint in view of the formidable advances in the area of metamaterials science and engineering, whereby propagation scenarios featuring highly inhomogeneous and anisotropic distributions of the constitutive parameters are becoming realistic.

Within this framework, transformation optics (TO) [1, 2] provides a powerful, systematic tool that can be applied to the modeling and design of sophisticated field manipulations [3]. The reader is referred to [4] for a recent roadmap addressing the state of the art in the field and outlining possible perspectives and future challenges, and to [5, 6] for examples of applications beyond electromagnetics.

One of the main challenges that this research field is facing is enlarging the class of material properties that can be encompassed. Starting from the inherent inhomogeneous and anisotropic properties entailed in the conventional TO formulation [1, 2], extensions to nonreciprocal, nonlinear, single-negative-parameter, reconfigurable scenarios have been proposed [7]–[13]. Here, we review some extensions to scenarios featuring spatial dispersion and spatially modulated gain and loss distributions, which are eliciting a growing attention in optics and photonics.

2 Transformation-Optics Extensions

2.1 Nonlocal: Spectral-Domain Formulation

Spatial-dispersion (nonlocal) effects imply convolutional operators that are most naturally dealt with in the spectral (wavevector) domain. Accordingly, in a series of recent studies [14]–[16], we put forward a spectral-domain extension of TO in the frequency-wavevector phase space. This enables the study and engineering of a variety of complex space-time dispersion effects, including unidirectional propagation, additional extraordinary waves, Dirac-cone-type singularities, frozen waves and degenerate band edges. Remarkably, our approach retains the intuitive geometrical interpretation which characterizes conventional (spatial-coordinate) TO, this time in terms of geometrical characteristics and deformations of the dispersion surfaces (e.g., center-symmetry, multi-valuedness, phase-space shifts, stationary points, etc.).

One of the most fascinating and challenging applications is harnessing spatial dispersion so as to attain desired *signal-processing* [17] or even *equation-solving* [18] capabilities. To this aim, we are working on a more general formulation relying on windowed (e.g., Gabor-type) spatial Fourier transforms, which grants access to the *space-wavevector* phase space. This should allow to unify under the same umbrella the conventional (local) and nonlocal TO formulations, yielding some tradeoff resolution in both the spatial and wavevector domains, and hence combining *spatial-routing* and *signal-processing* capabilities. As possible physical platforms, spatially variant lattices [19] appear potentially suitable and promising.

2.2 Non-Hermitian: Complex-Coordinate Formulation

Conventional TO formulations, based on *real-valued* coordinate transformations, inherently yield real-valued constitutive parameters, and therefore do not allow to model and manipulate *extrinsic* distributions of gain and loss. As we showed in [20, 21], these restrictions can be lifted by extending the coordinate transformations to a *complex* space (see also [22, 23] for related formulations). This extension enables the modeling and design of non-Hermitian systems characterized by spatial modulations of gain and loss [24], also providing new mathematical and physical insights. For instance, we were able to relate the appearance of distinctive features such as "exceptional points" (i.e., covalescence of multiple eigenstates) with the presence of discontinuities in the complex-coordinate mapping [20]. As for the physical interpretation, the "complexification" of spatial and spectral observables is actually not new in the electromagnetics community, and reliance can be made on well-established formalisms and concepts such as the *complex-source-point* (CSP) approach [25, 26] and leaky waves. For instance, we applied these formalisms to synthesize "transformation slabs" capable of manipulating Gaussian beams and leaky waves [21]. In their simplest form, these slabs comprise piecewise homogeneous, uniaxal (possibly with tilted optical axis) materials with constitutive parameters featuring both loss and gain along different directions.

3 Conclusions and Perspectives

In many electromagnetic scenarios of current practical interest, especially in connection with metamaterials science and engineering, nonlocal (spatial dispersion) and non-Hermitian (spatially modulated gain/loss distributions) are playing an increasingly important role. Conventional TO approaches based on real-valued, spatial-domain coordinate transformations are not able to handle these scenarios, but can be extended (via spectral-domain and complex-coordinate formulations) so as to encompass them. These extensions provide powerful and systematic frameworks for the study and engineering of wave propagation in such complex scenarios, while maintaining the insightful characteristics of conventional TO in terms of geometrical intuition. We are currently working on unifying these extensions under the broad umbrella of the space-wavevector phase-space.

References

[1] U. Leonhardt, "Optical Conformal Mapping," *Science*, **312**, 5781, June 2006, pp. 1777–1780, doi: 10.1126/science.1126493.

[2] J. Pendry, D. Schuring, and D. R. Smith, "Controlling Electromagnetic Fields," *Science*, **312**, 5781, June 2006, p. 1780–1782, doi: 10.1126/science.1125907.

[3] D. H. Werner and D. H. Kwon, *Transformation Electromagnetics and Metamaterials: Fundamental Principles and Applications*, Springer, Berlin, 2013.

[4] M. McCall *et al.*, "Roadmap on Transformation Optics," *Journal of Optics*, **20**, 6, June 2018, 063001, doi: 10.1088/2040-8986/aab976.

[5] M. Kadic, T. Bückmann, R. Schittny, and M. Wegener, "Metamaterials Beyond Electromagnetism," *Reports on Progress in Physics*, **76**, 12, December 2013, 126501, doi: 10.1088/0034-4885/76/12/126501.

[6] M. Moccia, G. Castaldi, S. Savo, Y. Sato, and V. Galdi, "Independent Manipulation of Heat and Electrical Current via Bifunctional Metamaterials," *Physical Review X*, **4**, 2, May 2014, 021025, doi:10.1103/PhysRevX.4.021025.

[7] L. Bergamin, "Generalized Transformation Optics from Triple Spacetime Metamaterials," *Physical Review A*, **78**, 4, October 2008, 043825, doi: 10.1103/PhysRevA.78.043825.

[8] L. Bergamin, P. Alitalo, and S. A. Tretyakov, "Nonlinear Transformation Optics and Engineering of the Kerr Effect," *Physical Review B*, **84**, 20, November 2001, 205103, doi: 10.1103/PhysRevB.84.205103.

[9] S. A. Cummer and R. T. Thompson, " Frequency Conversion by Exploiting Time in Transformation Optics," *Journal of Optics*, **13**, 2, February 2011, 024007, doi: 10.1088/2040-8978/13/2/024007.

[10] G. Castaldi, I. Gallina, V. Galdi, A. Alù, and N. Engheta, "Transformation-Optics Generalization of Tunnelling Effects in Bi-layers Made of Paired Pseudo-Epsilon-Negative/Mu-Negative Media," *Journal of Optics*, **13**, 2, February 2011, 024011, doi: 10.1088/2040-8978/13/2/024011.

[11] R. T. Thompson, S. A. Cummer, and J. Frauendiener, "A Completely Covariant Approach to Transformation Optics," *Journal of Optics*, **13**, 2, February 2011, 024008, doi: 10.1088/2040-8978/13/2/024008.

[12] O. Paul and M. Rahm, "Covariant Description of Transformation Optics in Nonlinear Media," Optics Express, **20**, 8, April 2012, pp. 8982—8997, doi: 10.1364/OE.20.008982.

[13] S. Savo, Y. Zhou, G. Castaldi, M. Moccia, V. Galdi, S. Ramanathan, and Y. Sato "Reconfigurable Anisotropy and Functional Transformations with VO2-Based Metamaterial Electric Circuits," *Physical Review B*, **91**, 13, April 2015, doi: 10.1103/PhysRevB.91.134105.

[14] G. Castaldi, V. Galdi, A. Alù, and N. Engheta, "Nonlocal Transformation Optics," *Physical Review Letters*, **108**, 6, February 2012, 063902, doi: 10.1103/PhysRevLett.108.063902.

[15] M. Moccia, G. Castaldi, V. Galdi, A. Alù, and N. Engheta, "Dispersion Engineering via Nonlocal Transformation Optics," *Optica*, **3**, 2, February 2016, pp. 179–188, doi: 10.1364/OPTICA.3.000179.

[16] M. Moccia, G. Castaldi, and V. Galdi, "Degenerate-Band-Edge Engineering Inspired by Nonlocal Transformation Optics," *EPJ Applied Metamaterials*, **3**, July 2016, 2, doi: 10.1051/epjam/2016003.

[17] A. Silva, F. Monticone, G. Castaldi, V. Galdi, A. Alù, and N. Engheta, "Performing Mathematical Operations with Metamaterials," *Science*, **343**,

6167, January 2014, pp. 160–163, doi: 10.1126/science.1242818.

[18] N. M. Estakhri, B. Edwards, and N. Engheta, "Inverse-Designed Metastructures that Solve Equations," *Science*, **363**, 6433, March 2019, pp. 1333–1338, doi: 10.1126/science.aaw2498.

[19] R. C. Rumpf, J. J. Pazos, J. L. Digaum, and S. M. Kuebler, "Spatially Variant Periodic Structures in Electromagnetics," *Philosophical Transactions of The Royal Society A*, **373**, 2049, August 2015, 20140359, doi: 10.1098/rsta.2014.0359.

[20] G. Castaldi, S. Savoia, V. Galdi, A. Alù, and N. Engheta, "PT Metamaterials via Complex-Coordinate Transformation Optics," *Physical Review Letters*, **110**, 17, April 2013, 173901, doi: 10.1103/PhysRevLett.110.173901.

[21] S. Savoia, G. Castaldi, and V. Galdi, "Complex-Coordinate Non-Hermitian Transformation Optics," *Journal of Optics*, **18**, 4, April 2016, 044027, doi: 10.1088/2040-8978/18/4/044027.

[22] B.-I. Popa and S. A. Cummer, "Complex Coordinates in Transformation Optics," *Physical Review A*, **84**, 6, December 2011, 063837, doi: 10.1103/PhysRevA.84.063837.

[23] H. Odabasi, K. Sainath, and F. L. Teixeira, "Launching and Controlling Gaussian Beams from Point Sources via Planar Transformation Media," *Physical Review B*, **97**, 7, February 2018, 075105, doi: 10.1103/PhysRevB.97.075105.

[24] L. Feng, R. El-Ganainy, and L. Ge, "Non-Hermitian Photonics Based on Parity-Time Symmetry," *Nature Photonics*, **11**, 12, January 2017, pp. 752–762, doi: 10.1038/s41566-017-0031-1

[25] G. A. Deschamps, "Gaussian Beams as a Bundle of Complex Rays," *Electronics Letters*, **7**, 23, November 1971, pp. 684–685, doi: 10.1049/el:19710467.

[26] L. B. Felsen, "Complex-Source-Point Solutions of the Field Equations and Their Relation to the Propagation and Scattering of Gaussian Beams," *Symposia Mathematica*, **18**, 1976, pp. 39–56.