High-Accuracy Numerical Scheme for Finite Difference on Staggered Grid

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The Finite-Difference Time-Domain (FDTD) method [1,2] has been used for computational electromagnetism for more than a half century. The FDTD method achieves second-order accuracy in both space (domain) and time by using the staggered Yee grid system [1] without temporary work arrays. Also, the Yee grid system enforces the divergence-free constraint for both electric and magnetic fields in multidimensions. However, the FDTD method cannot suppress numerical oscillations due to its second-order accuracy, which arise from both of a continuous profile with a large spatial gradient and a discontinuous profile. The former numerical oscillations are prevented by using a higher-order finite difference scheme for the first derivative [3]. However, the latter numerical oscillations are prevented by using a large artificial viscosity.

In the present study, we try to improve an accuracy of the numerical method for analyzing electromagnetic waves by using a one-dimensional wave (advection) equation and a semi-Lagrangian-type scheme. We transform the one-dimensional finite difference equation of the standard FTDT method [1] and derive a semi-Lagrangian-type time difference equation, which includes the first, second and third derivatives as shown below.

$$E_z^{t+\Delta t} = E_z^t + c^2 \Delta t \frac{\partial B_y^t}{\partial x} + \alpha_1 c^2 {\Delta t}^2 \frac{\partial^2 E_z^t}{\partial x^2} + \beta_1 c^4 {\Delta t}^3 \frac{\partial^3 B_y^t}{\partial x^3}. \quad (1)$$

$$B_y^{t+\Delta t} = B_y^t + \Delta t \frac{\partial E_z^t}{\partial x} + \alpha_2 c^2 {\Delta t}^2 \frac{\partial^2 B_y^t}{\partial x^2} + \beta_2 c^2 {\Delta t}^3 \frac{\partial^3 E_z^t}{\partial x^3}. \quad (2)$$

The standard FDTD method [1] corresponds to the coefficients $\alpha_1 = \alpha_2 = 1/2, \beta_1 = 1/4, \beta_2 = 0$ (or $\beta_1 = 0, \beta_2 = 1/4$). We add fourth derivative terms in these equations and try to find an appropriate combination of the coefficients that achieve non-oscillatory or high accuracy. We also try to extend the scheme to multidimensions.

References

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