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Pattern Synthesis With Closed-Form Array Response Control Algorithms

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Abstract

In this paper, the problem of array pattern synthesis is considered by developing two array response control algorithms. The devised two algorithms build on the basis of the accurate array response control (A^2RC) approach, and both of them have closed-form solutions and low computational complexities. More specifically, the first proposed algorithm realizes multi-point array response control from an any given weight vector, and the second algorithm modifies the first one by avoiding the possible beam axis shift. By applying the two closed-form array response control algorithms, an effective pattern synthesis approach is devised. Representative examples are presented to demonstrate the performance of the proposed algorithms in array response control and pattern synthesis.

1 Introduction

Array pattern synthesis has been widely applied in the fields like radar, communication and remote sensing [1–3]. Briefly speaking, array pattern synthesis is to find an appropriate weight vector that makes a beampattern satisfy some prescribed specifications.

Quite a number of approaches to pattern synthesis have been developed in the past few decades. For example, the Chebyshev weighting [4] gives an analytical solution to realize beampatterns with equal sidelobe levels, for uniformly spaced linear arrays. For the arbitrary arrays, global search methods are adopted in [5,6] to find desirable patterns, with high computation costs. The adaptive array theory is utilized in [7] to synthesize satisfactory patterns by assigning virtual interferences. Nevertheless, it is not clear how to specify the virtual interferences systematically. There are also some approaches developed to synthesize patterns with the help of convex optimization, see [8,9] for instance. Note that in the above approaches, the weight vector has to be completely redesigned even if only a slight change of the desired pattern is needed.

Recently, array pattern synthesis has been realized from an any initialized weight, by using the array response control algorithms. More specifically, an accurate array response control (A^2RC) algorithm is presented in [10] to control array response level of a single point. And a new pattern synthesis scheme is reported accordingly, by continuously applying A^2RC on one appropriately-selected angle. On the basis of A^2RC, the multiple-point accurate array response control (MA^2RC) algorithm and the modified MA^2RC (M^2A^2RC) algorithm are presented in [11] to, respectively, control the array responses of multiple point, and avoid the possible beam axis shift when carrying out the multi-point response level adjustment. Note that both MA^2RC and M^2A^2RC provide closed-form expressions, however, their computation costs are high especially for a large array.

To overcome the shortcomings of MA^2RC and M^2A^2RC, two computationally attractive array response algorithms are presented in this paper. The proposed two algorithms achieve the similar functions as MA^2RC and M^2A^2RC, respectively, but greatly lowering the computation costs. We apply the proposed two algorithms to pattern synthesis and give representative examples to illustrate the performances.

2 Preliminaries

2.1 Array Response Control Formulation

Without loss of generality and for the sake of clarity, we focus on herein the problem of one-dimensional response control. The extension to more complicated configurations is straight-forward. First of all, the array power response is expressed as

$$L(\theta,\theta_0) = |\mathbf{w}^{\mathrm{H}}\mathbf{a}(\theta)|^2/|\mathbf{w}^{\mathrm{H}}\mathbf{a}(\theta_0)|^2 \quad (1)$$

where $(\cdot)^{\mathrm{H}}$ denotes the conjugate transpose, $\mathbf{w}$ is the weight vector, θ_0 is the main beam axis, $\mathbf{a}(\theta)$ stands for the steering vector in direction θ. More exactly, we have

$$\mathbf{a}(\theta) = [g_1(\theta)e^{-j\omega\tau_1(\theta)},\cdots,g_N(\theta)e^{-j\omega\tau_N(\theta)}]^{\mathrm{T}} \quad (2)$$

where $(\cdot)^{\mathrm{T}}$ denotes the transpose operator, $j=\sqrt{-1}$ is the imaginary unit, $g_n(\theta)$ denotes the pattern of the nth element, $\tau_n(\theta)$ represents the time-delay between the nth element and the reference point, $n=1,\cdots,N$, ω denotes the operating frequency. The problem of array response control is to find an appropriate weight vector which makes the normalized response $L(\theta,\theta_0)$ meet certain specific requirements.

2.2 A^{2}RC Algorithm

A single-point array response control algorithm was presented in [10]. More specifically, for a given weight vector $\mathbf{w}_{k-1}$, an angle θ_k to be controlled and its desired level ρ_k, a new weight $\mathbf{w}_k$ is expressed in A^{2}RC algorithm as

$$\mathbf{w}_k = \mathbf{w}_{k-1} + \mu_k \mathbf{a}(\theta_k) \tag{3}$$

where the parameter μ_k depends on ρ_k, and can be analytically expressed as shown in Eqn.(52) of [10].

3 The Proposed Closed-Form Array Response Control Algorithms

The A^{2}RC algorithm can only adjust the response of a single angle each time. Moreover, it may lead to beam axis shift during the response control process. To overcome these drawbacks, we present two new array response control algorithms in this section, by extending A^{2}RC.

To make the following discussions clear, we give a weight $\mathbf{w}_{k-1}$, a set of pre-assigned angles $\theta_1, \theta_2, \cdots, \theta_Q$ and their desired levels $\rho_1, \rho_2, \cdots, \rho_Q$. Then, we consider how to find a weight $\mathbf{w}_k$ that satisfies

$$L_k(\theta_q, \theta_0) \triangleq \frac{|\mathbf{w}_k^{\mathrm{H}} \mathbf{a}(\theta_q)|^2}{|\mathbf{w}_k^{\mathrm{H}} \mathbf{a}(\theta_0)|^2} = \rho_q,\ q = 1, \cdots, Q. \tag{4}$$

In addition, we also study how to design a $\mathbf{w}_k$ satisfying (4) without leading to the beam axis shift. We assume that $\mathbf{a}(\theta_0), \mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_Q)$ are linearly independent and present the first algorithm next.

3.1 The First Algorithm

To realize the multi-point array response control task (4) as described earlier, we first calculate the weight vector $\breve{\mathbf{w}}_{k,q}$, which adjusts the response level of θ_q to be ρ_q, $q = 1, \cdots, Q$, respectively, by using the A^{2}RC Algorithm [10]. More specifically, for an given index $q = 1, \cdots, Q$, we express $\breve{\mathbf{w}}_{k,q}$ as

$$\breve{\mathbf{w}}_{k,q} = \mathbf{w}_{k-1} + \mu_{k,q} \mathbf{a}(\theta_q) \tag{5}$$

where $\mu_{k,q}$ can be analytically expressed by A^{2}RC and the resulting $\breve{\mathbf{w}}_{k,q}$ satisfies

$$|\breve{\mathbf{w}}_{k,q}^{\mathrm{H}} \mathbf{a}(\theta_q)|^2 / |\breve{\mathbf{w}}_{k,q}^{\mathrm{H}} \mathbf{a}(\theta_0)|^2 = \rho_q. \tag{6}$$

Once the weight vector $\breve{\mathbf{w}}_{k,q}$ in (5) is obtained, we scale it for later use as

$$\mathbf{w}_{k,q} = \breve{\mathbf{w}}_{k,q} / (\mathbf{a}^{\mathrm{H}}(\theta_0) \breve{\mathbf{w}}_{k,q}). \tag{7}$$

It is not hard to verify that the above resulting $\mathbf{w}_{k,q}$'s satisfy

$$\mathbf{w}_{k,q}^{\mathrm{H}} \mathbf{a}(\theta_0) = 1,\ q = 1, \cdots, Q \tag{8}$$

and then,

$$|\mathbf{w}_{k,q}^{\mathrm{H}} \mathbf{a}(\theta_q)|^2 = \rho_q,\ q = 1, \cdots, Q. \tag{9}$$

To find a single weight $\mathbf{w}_k$ that realizes the multiple-point array response control task (4), we propose to constraint the ultimate weight $\mathbf{w}_k$ by

$$\mathbf{w}_k^{\mathrm{H}} \mathbf{a}(\theta_q) = \mathbf{w}_{k,q}^{\mathrm{H}} \mathbf{a}(\theta_q),\ q = 1, \cdots, Q \tag{10a}$$

$$\mathbf{w}_k^{\mathrm{H}} \mathbf{a}(\theta_0) = 1. \tag{10b}$$

Recalling (8) and (9), one can verify that the resulting $\mathbf{w}_k$ realizes the response control task (4), if only (10) is true.

Since $\mathbf{w}_{k,q}$, $q = 1, \cdots, Q$, have been obtained by (7), one can compact (10) as

$$\mathbf{A}_k^{\mathrm{H}} \mathbf{w}_k = \mathbf{c}_k \tag{11}$$

where

$$\mathbf{A}_k = [\mathbf{a}(\theta_0), \mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_Q)] \in \mathbb{C}^{N \times (Q+1)} \tag{12a}$$

$$\mathbf{c}_k = \left[1, \mathbf{w}_{k,1}^{\mathrm{H}} \mathbf{a}(\theta_1), \cdots, \mathbf{w}_{k,Q}^{\mathrm{H}} \mathbf{a}(\theta_Q)\right]^{\mathrm{H}} \in \mathbb{C}^{Q+1}. \tag{12b}$$

Recalling that $\mathbf{A}_k^{\mathrm{H}}$ has a full row rank if $Q \leq N-1$, one solution of (11) is given by

$$\mathbf{w}_k = (\mathbf{A}_k^{\mathrm{H}})^{\dagger} \mathbf{c}_k = \mathbf{A}_k (\mathbf{A}_k^{\mathrm{H}} \mathbf{A}_k)^{-1} \mathbf{c}_k \tag{13}$$

where $(\cdot)^{\dagger}$ denotes the pseudo-inverse of a matrix.

3.2 The Second Algorithm

The above algorithm gives a closed-form solution to control array responses at multiple points. Nevertheless, it may lead to possible shift of the beam axis. In other words, the ultimate maximum value of pattern corresponds to $\mathbf{w}_k$ in (13) may appear in another angle rather than the desired beam axis θ_0.

In this subsection, the second algorithm is presented to control array responses of multiple angles, without leading to beam axis shift. More specifically, for a given previous weight $\mathbf{w}_{k-1}$, θ_q and its corresponding desired level ρ_q, $q = 1, \cdots, Q$, we consider how to select an appropriate weight vector $\mathbf{w}_k$ satisfying (4), and additionally,

$$\theta_0 = \arg\max_{\theta} |\mathbf{w}_k^{\mathrm{H}} \mathbf{a}(\theta)|. \tag{14}$$

To make the extra constraint (14) satisfied, we follow the idea in [11] and impose an additional derivative constraint as

$$\left.\frac{\partial P(\theta)}{\partial \theta}\right|_{\theta=\theta_0} = 0 \tag{15}$$

where $P(\theta) = \mathbf{w}_k^{\mathrm{H}}\mathbf{a}(\theta)\mathbf{a}^{\mathrm{H}}(\theta)\mathbf{w}_k$ denotes the array power response. Substitute the expression of $P(\theta)$ into (15), we have

$$\begin{aligned}\frac{\partial P(\theta)}{\partial \theta} &= \mathbf{w}_k^{\mathrm{H}}\frac{\partial \mathbf{a}(\theta)}{\partial \theta}\mathbf{a}^{\mathrm{H}}(\theta)\mathbf{w}_k + \mathbf{w}_k^{\mathrm{H}}\mathbf{a}(\theta)\frac{\partial \mathbf{a}^{\mathrm{H}}(\theta)}{\partial \theta}\mathbf{w}_k \\ &= 2\mathrm{Re}\left[\mathbf{w}_k^{\mathrm{H}}\frac{\partial \mathbf{a}(\theta)}{\partial \theta}\mathbf{a}^{\mathrm{H}}(\theta)\mathbf{w}_k\right] \end{aligned} \tag{16}$$

and further

$$\left.\frac{\partial P(\theta)}{\partial \theta}\right|_{\theta=\theta_0} = 2\mathrm{Re}\left[\mathbf{w}_k^{\mathrm{H}}\mathbf{d}(\theta_0)\mathbf{a}^{\mathrm{H}}(\theta_0)\mathbf{w}_k\right] \tag{17}$$

where $\mathbf{d}(\theta_0)$ is defined as

$$\mathbf{d}(\theta_0) \triangleq \left.\frac{\partial \mathbf{a}(\theta)}{\partial \theta}\right|_{\theta=\theta_0}. \tag{18}$$

Then, to realize multi-point array response control without the beam axis shift, we will find a $\mathbf{w}_k$ such that

$$\frac{|\mathbf{w}_k^{\mathrm{H}}\mathbf{a}(\theta_q)|^2}{|\mathbf{w}_k^{\mathrm{H}}\mathbf{a}(\theta_0)|^2} = \rho_q,\ q = 1, \cdots, Q \tag{19a}$$

$$\Re\left[\mathbf{w}_k^{\mathrm{H}}\mathbf{d}(\theta_0)\mathbf{a}^{\mathrm{H}}(\theta_0)\mathbf{w}_k\right] = 0. \tag{19b}$$

In addition, to make the following discussion meaningful, we assume that $\mathbf{a}(\theta_0), \mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_Q)$ and $\mathbf{d}(\theta_0)$ are linearly independent.

Similar to the first algorithm presented in the preceding subsection, for a given weight $\mathbf{w}_{k-1}$, θ_q and its desired response level ρ_q, $q = 1, \cdots, Q$, we first calculate the weight vector $\mathbf{w}_{k,q}$ in (7) by applying the A^{2}RC Algorithm [10] and then conducting a normalization operator, see (5) and (7), respectively, for details. It has been known that the resulting $\mathbf{w}_{k,q}$, $q = 1, \cdots, Q$, satisfy both (8) and (9). Then, a simple way to make (19) hold true is to find a weight $\mathbf{w}_k$ satisfying

$$\mathbf{w}_k^{\mathrm{H}}\mathbf{a}(\theta_0) = 1 \tag{20a}$$

$$\mathbf{w}_k^{\mathrm{H}}\mathbf{a}(\theta_q) = \mathbf{w}_{k,q}^{\mathrm{H}}\mathbf{a}(\theta_q),\ q = 1, \cdots, Q \tag{20b}$$

$$\Re\left[\mathbf{w}_k^{\mathrm{H}}\mathbf{d}(\theta_0)\right] = 0. \tag{20c}$$

Comparing to (10) in the preceding subsection, one can see that an extra constraint (20c) is added above to avoid the possible beam axis shift.

To further find a closed-form solution of $\mathbf{w}_k$ satisfying (20), we define

$$\widetilde{\mathbf{v}} \triangleq \left[\Re(\mathbf{v}^{\mathrm{T}}) \quad \Im(\mathbf{v}^{\mathrm{T}})\right]^{\mathrm{T}}. \tag{21}$$

Then, it can be readily found that

$$\begin{aligned}\mathbf{w}_k^{\mathrm{H}}\mathbf{a}(\theta) &= \left[\Re(\mathbf{w}_k^{\mathrm{T}}) \quad \Im(\mathbf{w}_k^{\mathrm{T}})\right]\left[\begin{pmatrix}\Re[\mathbf{a}(\theta)] \\ \Im[\mathbf{a}(\theta)]\end{pmatrix} + j\begin{pmatrix}\Im[\mathbf{a}(\theta)] \\ -\Re[\mathbf{a}(\theta)]\end{pmatrix}\right] \\ &= \widetilde{\mathbf{w}}_k^{\mathrm{T}}\left[\widetilde{\mathbf{a}}(\theta) + j\boldsymbol{\Upsilon}\widetilde{\mathbf{a}}(\theta)\right]\end{aligned} \tag{22}$$

where $\boldsymbol{\Upsilon}$ is defined as

$$\boldsymbol{\Upsilon} \triangleq \begin{bmatrix} & \mathbf{I}_N \\ -\mathbf{I}_N & \end{bmatrix} \in \mathbb{R}^{2N\times 2N}. \tag{23}$$

In addition, one can verify that

$$\Re\left[\mathbf{w}_k^{\mathrm{H}}\mathbf{d}(\theta_0)\right] = \widetilde{\mathbf{w}}_k^{\mathrm{T}}\widetilde{\mathbf{d}}(\theta_0). \tag{24}$$

According to (22) and (24), (20) can be equivalently expressed as

$$\widetilde{\mathbf{w}}_k^{\mathrm{T}}\widetilde{\mathbf{a}}(\theta_0) = 1 \tag{25a}$$

$$\widetilde{\mathbf{w}}_k^{\mathrm{T}}\boldsymbol{\Upsilon}\widetilde{\mathbf{a}}(\theta_0) = 0 \tag{25b}$$

$$\widetilde{\mathbf{w}}_k^{\mathrm{T}}\widetilde{\mathbf{a}}(\theta_q) = \widetilde{\mathbf{w}}_{k,q}^{\mathrm{T}}\widetilde{\mathbf{a}}(\theta_q),\ q = 1, \cdots, Q \tag{25c}$$

$$\widetilde{\mathbf{w}}_k^{\mathrm{T}}\boldsymbol{\Upsilon}\widetilde{\mathbf{a}}(\theta_q) = \widetilde{\mathbf{w}}_{k,q}^{\mathrm{T}}\boldsymbol{\Upsilon}\widetilde{\mathbf{a}}(\theta_q),\ q = 1, \cdots, Q \tag{25d}$$

$$\widetilde{\mathbf{w}}_k^{\mathrm{T}}\widetilde{\mathbf{d}}(\theta_0) = 0. \tag{25e}$$

A compact version of (25) is

$$\mathbf{Z}_k^{\mathrm{T}}\widetilde{\mathbf{w}}_k = \mathbf{g}_k \tag{26}$$

where $\mathbf{Z}_k$ and $\mathbf{g}_k$ are given in (27) on the top of next page. The linear equation (26) is consistent, provided that $\mathbf{a}(\theta_0), \mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_Q)$ and $\mathbf{d}(\theta_0)$ are linearly independent. Moreover, one solution of (26) can be analytically expressed as

$$\widetilde{\mathbf{w}}_k = (\mathbf{Z}_k^{\mathrm{T}})^{\dagger}\mathbf{g}_k = \mathbf{Z}_k(\mathbf{Z}_k^{\mathrm{T}}\mathbf{Z}_k)^{-1}\mathbf{g}_k. \tag{28}$$

Once the above weight vector $\widetilde{\mathbf{w}}_k$ is obtained, we can reformulate it to complex domain and express the ultimate $\mathbf{w}_k$ as

$$\mathbf{w}_k = [\mathbf{I}_N,\ j\mathbf{I}_N]\,\widetilde{\mathbf{w}}_k. \tag{29}$$

It is not hard to validate that the resulting weight vector $\mathbf{w}_k$ in (29) satisfies (19). In other words, $\mathbf{w}_k$ can precisely control array response levels at multiple points, without leading to the possible beam axis shift.

4 Pattern Synthesis Using the Proposed Algorithms

The applications of the proposed two algorithms to pattern synthesis will be discussed in this section. The general strategy herein shares a similar concept of pattern synthesis using MA^{2}RC or M^{2}A^{2}RC in [11], however, reduces the computation complexity. Specifically, for an given weight vector $\mathbf{w}_0$ (can be freely initialized), multiple angles are first determined according to the current pattern and the desired one (denoted by $L_d(\theta)$). These angles can be in either sidelobe region or mainlobe region. For sidelobe synthesis, we select several peak angles where the response deviations (from the desired level) are relatively large. For mainlobe synthesis, few discrete angles where the responses deviate large from the desired ones are chosen. More details about the angle selection see [10] and [11]. Once those angles have been picked out, the proposed two algorithms can be utilized to adjust the corresponding responses to their desired values. This step is iteratively carried out until the response is satisfactorily synthesized.

$$\mathbf{Z}_k = \left[\widetilde{\mathbf{a}}(\theta_0), \boldsymbol{\Upsilon}\widetilde{\mathbf{a}}(\theta_0), \widetilde{\mathbf{a}}(\theta_1), \boldsymbol{\Upsilon}\widetilde{\mathbf{a}}(\theta_1), \cdots, \widetilde{\mathbf{a}}(\theta_Q), \boldsymbol{\Upsilon}\widetilde{\mathbf{a}}(\theta_Q), \widetilde{\mathbf{d}}(\theta_0)\right] \in \mathbb{R}^{2N\times(2Q+3)} \tag{27a}$$

$$\mathbf{g}_k = \left[1, 0, \widetilde{\mathbf{w}}_{k,1}^{\mathrm{T}}\widetilde{\mathbf{a}}(\theta_1), \widetilde{\mathbf{w}}_{k,1}^{\mathrm{T}}\boldsymbol{\Upsilon}\widetilde{\mathbf{a}}(\theta_1), \cdots, \widetilde{\mathbf{w}}_{k,Q}^{\mathrm{T}}\widetilde{\mathbf{a}}(\theta_Q), \widetilde{\mathbf{w}}_{k,Q}^{\mathrm{T}}\boldsymbol{\Upsilon}\widetilde{\mathbf{a}}(\theta_Q), 0\right]^{\mathrm{T}} \in \mathbb{R}^{2Q+3} \tag{27b}$$

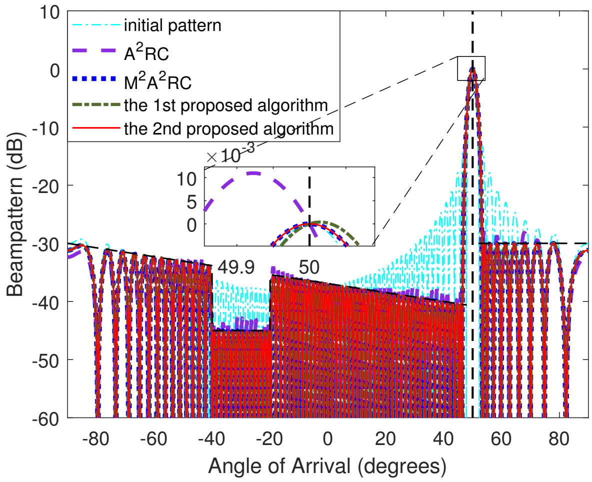

Figure 1. Comparison of the synthesized patterns.

5 Numerical Results

In this section, we present an example of pattern synthesis by using our array response control algorithms. More specifically, we consider a linearly half-wavelength-spaced array with $N = 80$ isotropic elements and take $\theta_0 = 50°$. The desired beampattern has a nonuniform sidelobes. The synthesis procedures are started with the initial weight vector $\mathbf{w}_0 = \mathbf{a}(\theta_0)$.

For comparison purpose, Fig. 1 shows the resulting patterns of the A^2RC algorithm in [10], the M^2A^2RC algorithm in [11] and our proposed two algorithms. One can see that both A^2RC and the first algorithm lead to a beam axis shift. When testing the result of M^2A^2RC, observations show that it results the same beampattern as the second proposed algorithm, and beam axis shift are avoided for both.

6 Conclusions

In this paper, we have presented two array response control algorithms. The proposed two algorithms build on the foundation of the existing A^2RC algorithm and can control array responses of multiple angles precisely as expected with closed-form solutions. Moreover, the devised two algorithms are computationally attractive, and the second one avoids the possible beam axis shift when adjusting the array response levels. We have also applied the proposed array response control algorithms to realize array pattern synthesis. Representative examples have been illustrated to show the effectivenesses and the superiorities of the proposed algorithms.

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