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Strong Fluctuation Theories of Tatarskii

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Physics-based models for remote sensing often involve wave equations with randomly fluctuating wave numbers. Early works in this area were based on perturbation methods. However, such approaches are applicable only when the field fluctuations are very weak. To overcome this limitation, Tatarskii developed two techniques, which are widely used by the remote sensing community.

One is based on renormalization using Feynman diagram representation. Tatarskii derived equations of first and second moments of wave functions, and showed that they enlarge the validity domain of perturbation-based results. Furthermore, he connected the results of first and second moments with the optical theorem. He hence showed that this development leads to the radiative transfer equation. This is one of the first derivation of the phenomenological radiative transfer equation starting from wave equations. There are several assumptions embedded in this result: wavelength should be much smaller than the correlation length of refractive index, which in turn must be much smaller than mean free path. The refractive index fluctuations should be a stationary Gaussian process.

The second technique of Tatarskii is based on the parabolic equation model. This model is meant for situations when wave propagation is predominantly in the direction in which it is launched. This implies negligible backscatter. In addition to this, the refractive index fluctuations are assumed (a) to obey stationary Gaussian statistics, and (b) are delta correlated along the direction of propagation. Using this model, Tatarskii proceeded to derive closed equations for wave-function-moments of any order. Note that no explicit assumption of small refractive index fluctuations is made in this approach.

In our paper we revisit this problem by taking a different approach. We first identify the main parameters of the problem and introduce reference scales. We employ asymptotic analysis and obtain results which are essentially similar to those obtained by Tatarskii. However, our conditions are different and more unified, which lead us to a different interpretation of the results. More importantly, our results provide a clearer definition of the validity regimes of the results of Tatarskii.