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Reflectivity, Emissivity, and Brightness from Multilayered Soil with Linearly Varying Permittivity at P- and L- bands

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Abstract

Passive sensing of root zone soil moisture is theoretically investigated using P and L band frequencies. The soil is considered to be a stratified medium. The reflectivity from the soil surface is found by considering the soil to be composed of a finely layered structure with layer widths that are small compared to a wavelength. A well-known transmission line algorithm is then used to compute the reflectivity at the surface. Emissivity is obtained as (1 – reflectivity). It assumed that the soil is at a constant temperature of 300° K when determining the brightness temperature. A soil moisture profile consisting of two homogeneous layers separated by a layer of linearly varying soil moisture is investigated. The study is made at an incident angle of 40^{O} for horizontal polarization at 0.8 and 1.4 GHz. Two variations of the basic profile are considered: a transition from a layer of lesser moisture to a layer of greater moisture and the opposite. In both cases the P band signal shows greater penetration than the L band case. There is greater penetration in the transition to drier soils. In both cases the L band signals show sensitivity to only the upper surface of the soil as expected.

1 Introduction

Remote sensing of soil moisture is one of the major applications of radar and radiometer technologies. SMAP and SMOS, L band passive remote sensing satellites are being used in the measurement of surface soil moisture under various types of vegetation cover ([1], [2]). There is a need, however, to provide soil moisture measurements at deeper soil depths for land surface hydrology and agricultural applications. Brightness temperatures at P and L band frequencies can be used to estimate a soil moisture profile and provide a mechanism to monitor root-zone soil moisture [3].

The soil moisture profile to depths of tens of centimeters is a key variable of controlling the hydrologic partitioning over land surfaces within root-zone depths. The soil moisture depth gradients can undergo significant variations and are determined by surface hydrologic fluxes, drainage and transpiration by deep-rooted vegetation. In this paper some of these depth-dependent soil moisture variations will be modeled by a dielectric profile which is a linear function of volumetric soil moisture content. The real part of the dielectric constant can range from 3 for relatively dry soil to about 17 (P-band) and 15 (L-band) for wet soil. Therefore, reflection coefficients at two frequency bands can vary significantly depending on the soil moisture regime.

In this paper, reflectivity, emissivity and brightness temperature are investigated from a layered medium whose dielectric profile is specified in eq. (1-1). The profile consists of a homogeneous layer under the soil surface. Below this homogeneous layer is a layer whose dielectric constant varies linearly with depth. Underneath this linear layer lies a homogeneous half space. Solutions for both polarizations and for arbitrary incidence angles are obtained by using the transmission line method. Two profiles will be examined: first, the case where the soil moisture increases with depth and the second case where it decreases with depth.

2 Problem Definition

As shown in Fig. 1, a plane wave is incident on a multilayered medium from free space at an angle θ_0 with respect to the z axis. The incident wave is polarized either horizontally or vertically. Here horizontal and vertical polarization imply that the electric or magnetic field is parallel to the y axis, respectively. The multilayered medium includes an inhomogeneous layer, which lies between the $z=-D_1$ and the $z=-D_2$ interfaces; it has a linear variation of the dielectric constant. The dielectric constant profile is continuous at $z=-D_1$ and $z=-D_2$. The medium above $z=-D_1$ is a homogeneous layer with relative dielectric constant $\varepsilon(-D_1)$ and thickness D_1. The medium below $z=-D_2$ is a homogeneous half space with relative dielectric constant $\varepsilon(-D_2)$. A time dependence of $\exp(j\omega t)$ is assumed. The permeability of all media is μ_0. In Fig. 1 the media have been labeled I, II, III, and IV, and the relative dielectric variation is written as

$$\varepsilon(z)=\begin{cases}1, & z>0\\ \varepsilon'(-D_1)-i\varepsilon''(-D_1), & z\geq -D_1\\ \varepsilon'(-D_1)(1+a'z)-i\varepsilon''(-D_1)(1+a''z), & -D_2\leq z\leq -D_1\\ \varepsilon'(-D_2)-i\varepsilon''(-D_2), & -D_2\geq z\end{cases} \quad . \text{(1-1)}$$

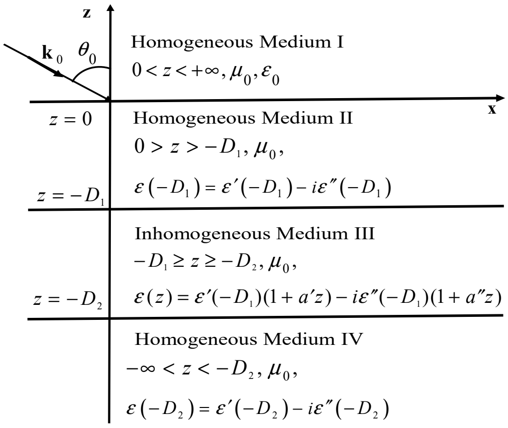

Figure 1. Schematic diagram of the problem.

In eq. (1), $\varepsilon(-D_1) = \varepsilon'(-D_1) - i\varepsilon''(-D_1)$ is the relative dielectric constant of medium II; $\varepsilon(-D_2) = \varepsilon'(-D_2) - i\varepsilon''(-D_2)$ is the relative dielectric constant of medium IV; a' and a'' are, respectively, the slope parameters of the real part and imaginary part for the function $\varepsilon(z)$ in medium III, which are written by

$$a' = \frac{1}{\varepsilon'(-D_1)} \times \frac{\varepsilon'(-D_2) - \varepsilon'(-D_1)}{-(D_2 - D_1)} \tag{1-2}$$

$$a'' = \frac{1}{\varepsilon''(-D_1)} \times \frac{\varepsilon''(-D_2) - \varepsilon''(-D_1)}{-(D_2 - D_1)}. \tag{1-3}$$

3 Derivation for Reflectivity and Emissivity

Any stratified inhomogeneous dielectric medium can be accurately modeled as a stack of piecewise homogeneous thin layers with the thickness of each layer much smaller than wavelength [4]. Therefore, the soil layer as given in Fig. 1 is discretized into a stack of homogeneous thin layers. The reflection and transmission coefficients are then obtained for each individual homogeneous thin layer. Finally, by applying the transmission line method [4], the generalized reflection and transmission coefficients of a multilayered medium are computed by recursively cascading the reflection and transmission coefficients of each thin layer from the bottom interface to the top interface. The generalized reflection and transmission coefficients at $z = -d_{n+1}$ are given by [5]

$$\tilde{R}^p_{n,n+1} = R^p_{n,n+1} + \frac{\left[1 - \left(R^p_{n,n+1}\right)^2\right] \tilde{R}^p_{n+1,n+2} \phi^2_{n+1,n+2}}{1 + R^p_{n,n+1} \tilde{R}^p_{n+1,n+2} \phi^2_{n+1,n+2}} \tag{2-1}$$

$$\tilde{T}^p_{n,n+1} = \frac{T^p_{n+1,n+2} \tilde{R}^p_{n+1,n+2} T^p_{n,n+1} \phi_{n+1,n+2}}{1 + R^p_{n,n+1} \tilde{R}^p_{n+1,n+2} \phi_{n+1,n+2}{}^2}. \tag{2-2}$$

In eq. (2), $R^p_{n,n+1}$ and $T^p_{n,n+1}$ are the Fresnel reflection and transmission coefficient of the individual thin layer, where the subscript n, $n+1$ denotes the interface between the n-th layer and the $n+1$-th layer; $\phi_{n+1,n+2} = e^{-jk_{(n+1)z}\Delta d}$, where $k_{(n+1)z}$ is the z component of the wave-number for $n+1$-th layer, and Δd is the thickness of each thin layer which is set to be uniform for all thin layers, $\Delta d = d_{n+1} - d_n$ with $1 \leq n \leq N$. The first layer between n=0 and n=1 has a width D_1. The superscript p denotes horizontal and vertical polarization.

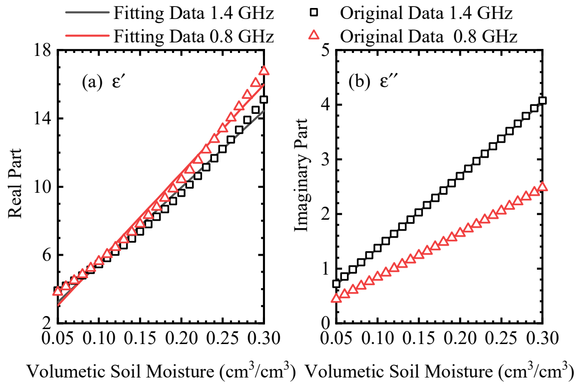

Figure 2. Semi-empirical model data and their fitting curves for (a) real part and (b) imaginary part.

For the interface $z = 0$, the generalized reflection and transmission of multilayered layer $\tilde{R}^p_{0,1}$ is obtained. With $\tilde{R}^p_{0,1}$, reflectivity, $\Re$, and emissivity, E, from the multilayered medium are given by the following formulas:

$$\Re = \left|\tilde{R}^p_{0,1}\right|^2 \tag{3-1}$$

$$\mathrm{E} = 1 - \Re. \tag{3-2}$$

4 Dielectric Constant Function

In this paper, Dobson's semi-empirical model [6]-[8] is used to determine the dielectric constant of soil with different values of volumetric soil moisture (Mv). A soil having a silty clay texture has been chosen for this study. Fig. 2 shows Dobson semi-empirical model data (dotted line) and their linear fitting curves (straight line) as function of Mv. This work is performed using the Matlab Curve Fitting tool. In the fitting curves, the real and imaginary parts of dielectric constant are written as

$$\varepsilon' = a_r \times m_v + b_r \tag{4-1}$$

$$\varepsilon'' = a_g \times m_v + b_g. \tag{4-2}$$

In this study, Mv is varied from 0.05 cm^3/cm^3 to 0.30 cm^3/cm^3, which is typically soil from dry to wet. With this choice, a_r, b_r, a_g, and b_g are assigned, respectively, to 55.7, 5.13, 8.19, and 0.833 for 0.8 GHz, and they are, respectively, 48.3, 5.04, 13.5, and 1.36 for 1.4 GHz. The data for dielectric constant given in Fig. 2 will be used in sections 5 and 6.

5 Skin Depth

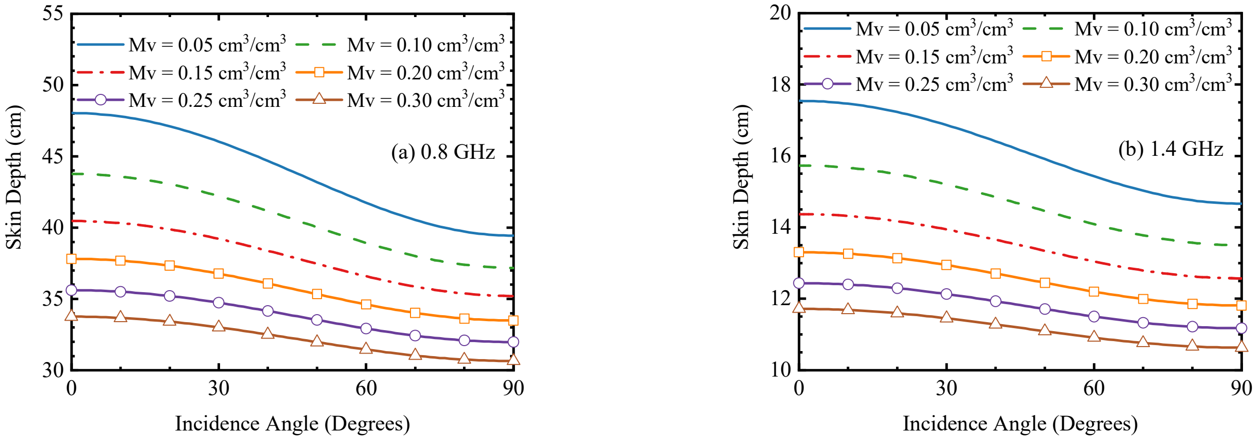

Figure 3. Skin depth as a function of incidence angle for different values of Mv, which corresponding dielectric constants are obtained from Figure 2; (a) 0.8 GHz; (b) 1.4 GHz.

Skin depth is a key parameter for investigating the penetration of EM waves in the soil; it represents the distance from the interface at which the relative signal level attenuates to e^{-1}. It is determined by the inverse of the imaginary part of the EM wavenumber. Fig. 3 shows skin depth as a function of incidence angle at 0.8 GHz and 1.4 GHz for a medium with Mv ranging from 0.05 cm^3/cm^3 to 0.30 cm^3/cm^3 , where the dielectric constants are obtained from Fig. 2. From Figs. 3 (a) and (b), it is seen that skin depth decreases with the increasing values of Mv and incidence angle. In addition, the skin depth at 0.8 GHz is significantly larger than it is at 1.4 GHz for fixed incidence angle and Mv.

6 Reflectivity and Emissivity from Soil

In order to provide more information for the moisture retrieval depth at 0.8 GHz and 1.4 GHz, reflectivity and emissivity from an inhomogeneous soil layer are calculated by using eq. (3). The brightness temperature is computed from the emissivity using a soil temperature of 300° K. The number of thin layers is set to 50, which meets the condition of $\Delta d \le \lambda/10$ for approximating the dielectric constant profile. The dielectric constant involved in the following simulations are obtained from Fig. 2.

First, Medium III is considered as having a linearly increasing moisture profile. This case can model an irrigated farmland with water slowly evaporating from soil surface. The considered moisture profile is shown in Fig. 4 (a). The reflectivity, emissivity and brightness temperature are plotted in Fig. 4 (b) as functions of the depth of Medium III D for H-Pol. The depth of Medium II is set to D_1=10 cm such that the L band wave decays while the P band wave does not decay as much based on the skin depths given in Fig. 3.

From Fig. 4 it is seen that the reflectivity values for both frequencies basically decrease as D becomes larger. In addition, the sensitivity of 0.8 GHz reflectivity to the variation of Mv in Medium III is stronger than that of 1.4 GHz reflectivity for dry soil. As the soil moisture of Medium II increases to moist and wet, the skin depth of the L band signal has almost no penetration into medium III. As a result, the 1.4 GHz reflectivity cannot sense the variation of Mv in Medium III. Conversely, the 0.8 GHz reflectivity can still give a significant response to the variation of Mv in Medium III. This implies that the P band wave is better for sensing the variation on Mv of soil at the deeper depth than the L band wave.

For the second case, Medium III has a linearly decreasing moisture profile. This case might occur when the soil has been dry for some time and then a rain event occurs. After the rain, the water slowly diffuses into the soil. The considered moisture profile is shown in Fig. 5 (a). The reflectivity, emissivity and brightness temperature are plotted in Fig. 5 (b) as functions of the depth of medium III D for H-Pol. Here again it is seen that the L band signal has very little penetration into region III. The P band signal, on the other hand, penetrates into region III. In addition, there appears to be a resonant effect that has increasing magnitude as the layer near the air-soil surface becomes wetter. As D increases, the effective layer width increases and the reflectivity goes up and out of resonance.

The two cases considered above both show that the P band wave can sense soil moisture variations at deeper depths than L band wave. It is interesting to note, however, that the P band wave appears to penetrate more deeply for the case of a wet upper layer over a dry half space below. Results have also been obtained for vertical polarized waves but have not been presented because of space restrictions.

7 Conclusions

In this paper, the reflectivity over a multilayered soil having linearly varying permittivity is derived by using the transmission line method. The permittivity profile of the

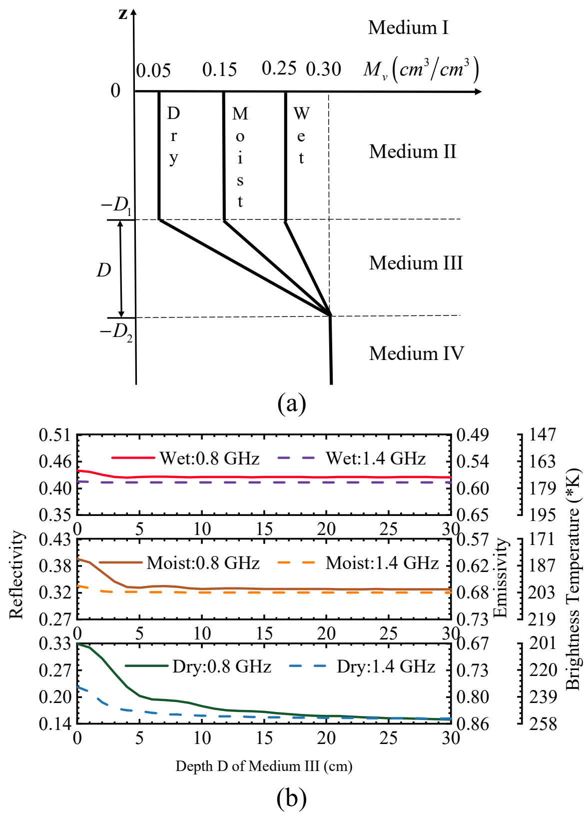

Figure 4. (a) the moisture profile of multilayered soil; (b) comparison of the reflectivity, emissivity, and brightness temperature versus the depth of Medium III ($D = D_2 - D_1$) at H-Pol, where D_1 =10 cm.

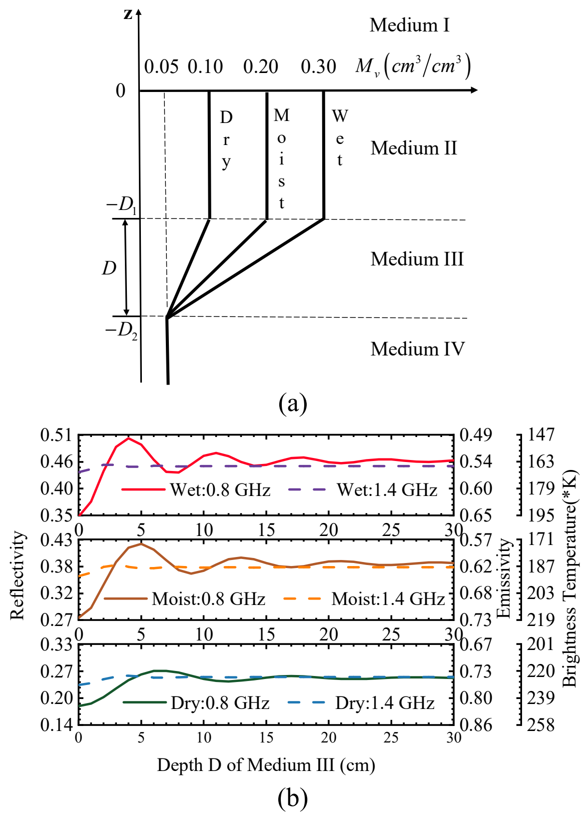

Figure 5. (a) the moisture profile of multilayered soil; (b) comparison of the reflectivity, emissivity, and brightness temperature versus the depth of Medium III ($D = D_2 - D_1$) at H-Pol, where D_1 =10 cm.

inhomogeneous soil is modeled as a linear function of Mv in the range of 0.05 cm^3/cm^3 to 0.30 cm^3/cm^3 . The sensitivities of reflectivity, emissivity, and brightness temperature are examined to the Mv and depth of soil layer at the frequencies of 0.8 GHz and 1.4 GHz and incidence angle of 40 degrees. For the soils with linearly increasing and decreasing moisture profile, the results demonstrate that the P band wave is better for sensing variations of soil Mv at deeper depths than the L band wave. In future work, soil moisture profiles are not restricted to a linear variation as an area for further investigation.

7 References

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