Cross-correlation index and multiple-access performance of spreading codes

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Extended Abstract. Spread spectrum technique is an invaluable communication modality that was developed originally for the military, but later became an important wireless communication technique in the general world. It is one of the pioneering access methods in mobile telephony, and it remained an important technique in the global positioning system (GPS) as well as in other position-location and navigation applications. Other areas of application include digital terrestrial television, anti-jam acoustic underwater communication, tomographic imaging and signal encryption. It is well known that any spread-spectrum system has an upper limit on the number of users that can be tolerated. However, the fraction of a code set that is available for use before the onset of system saturation is not well known. This work provides an answer to this problem. This work involves the development of a concept of cross-correlation index for the determination of the system performance limit with reference to its spreading sequences. Starting from basic principles, it can be shown that output signal $z_1$ for a reference user-1 in a multi-user system is given by:

$$z_1(m) = \frac{A}{N} \int_{mT_b}^{(m+1)T_b} \left( \sum_{l=1}^{L} \beta_l e^{j\gamma_l} c(t - \tau_{1l}) b_1(t) \cos(\omega_c t - \theta_{1l}) \right) c_1(t) \cos(\omega_c t - \theta_{1l}) dt
+ \frac{A}{N} \int_{mT_b}^{(m+1)T_b} \left( \sum_{k=2}^{K} \sum_{l=1}^{L} \beta_{kl} e^{j\gamma_{kl}} c_k(t - \tau_{kl}) b_k(t) \cos(\omega_c t - \theta_{kl}) \right) c_1(t - \tau_{1l}) \cos(\omega_c t - \theta_{kl}) dt
+ \int_{mT_b}^{(m+1)T_b} n(t) c_1(t - \tau_{kl}) \cos(\omega_c t - \theta_{kl}) dt = z_{11} + z_{12} + z_{13}$$

Proceeding from this, it can be shown that the interference term $z_{12}$ will reduce to (2), to give cross-correlation index $D_{1k}$ defined by (3):

$$z_{12} = \pm \frac{A}{2N} \sum_{k=2}^{K} \sum_{l=1}^{L} \beta_{kl} e^{j\gamma_{kl}} \int_{mT_b}^{(m+1)T_b} \tilde{d}_{1k} dt$$,

$$D_{1k} = \pm \frac{A}{2N} \sum_{k=2}^{K} \int_{mT_b}^{(m+1)T_b} \tilde{d}_{1k} dt$$,

where $d_{1k} = c_1(t - \tau_{1l}) c_k(t - \tau_{kl})$, the cross-correlation between the code sequence for user-1 and user-$k$. To validate the theory, graph of cross-correlation index for a set of Gold codes were generated by direct implementation of (3). The result was compared with parametric curves for the system bit-error-rate (BER) performance involving transmission of QPSK symbols averaged over about a million samples. The graph (Figure 1) of the cross-correlation index show that cross-over point for autocorrelation index and cross-correlation index occur at about 30 users. Beyond this point, magnitude of interference becomes stronger than the strength of the transmitted signal. In essence, the system is expected to saturate when the number of users goes beyond 30. These results agree with software simulations (Figure 2) obtained for the system BER performance. This work is part of a larger study. Related work [1-3] carried out for some other sets of codes confirm that the cross-correlation index is a simple but powerful tool for predicting the system loading capacity.

Figure 1. Cross-correlation index for a set of 1023-chip Gold codes

Figure 2. The system bit-error-rate performance