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Sampling approach for the discretization of scattering operator in inhomogeneous medium

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Abstract

In this paper, a subsurface imaging problem is addressed. In particular, the focus here is to determine a strategy to collect the spatial data allowing to reduce the number of measurement points compared to more common approach. The idea is to sample the scattered field in order to leave unchanged the more significant singular values of the corresponding scattering operator. The proposed strategy results into a non-uniform arrangement of measurement points.

1 Introduction

Subsurface imaging entails dealing with an inverse scattering problem in which the targets are embedded within an inhomogeneous background medium. As such, it is relevant in a number of applicative contexts that range from non-destructive testing to geophysical prospecting, from buried-object detection to mine detection [1], etc.

As it is well known, electromagnetic inverse scattering problems require the inversion of a non-linear and ill-posed mathematical relationship. However, if quantitative reconstructions are not required, the problem can be drastically simplified by linearising the scattering equations, for example by using the Born approximation [2]-[6]. Accordingly, the problem can be recast as the inversion of a linearized scattering operator. While under ideal configuration data can be continuously collected over the observation domain (OD), in practical cases, they can be collected only on a finite and discrete set of points within OD. Accordingly, the arising problem is how to collect the field data in order to approximate at some extent the mathematical properties of the continuous model.

A similar aim was addressed in [7] for an homogeneous medium and in [8] for an inhomogeneous one. In these papers, the new sampling strategy was derived by determining the data positions in order to approximate the ideal point-spread function of a migration-like inversion algorithm. On contrary, here, we don't consider any particular inversion scheme.

Since the compactness of the scattering operator, the singular values goes to zero as their index increases. This entails that only the more significant singular values are required to characterize both the forward and the inverse problem [9]- [11]. In this paper, a strategy of collecting the data in order to leave almost unchanged the more significant singular values of the continuous operator is proposed within the framework of subsurface imaging. In order to do that, the sampling approach introduced in [12] is exploited. The latter allows to determine an equivalent discrete eigenvalue problem sharing the same eigenvalues of the continuous operator and, indirectly, suggests a sampling series to expand the data space and how many samples are required to represent the field. However, while in far-zone such an approach can be applied, in near-zone arguments presented in [12] are less fruitful. This is because in near-zone the kernel of the scattering operator does not meet the mathematical properties invoked in [12]. To overcome this drawback, it has been recently shown, for a homogeneous background medium, that by introducing an appropriate transformation of the observation variable, the kernel function can be recast as a quasi band-limited function [13] and [14]. This allowed to get a rule to sample the field that returns a non-uniformly distribution for the sampling points. Here, such an approach is extended to subsurface imaging scenario. In particular, in order to keep notation simple, a scalar two dimensional configuration is considered, where the investigation and the observation domains are parallel bounded strips separated by a distance greater than the wavelength. In particular, a multi-monostatic measurement configuration is exploited.

2 Problem description

Consider the 2D scalar scattering problem sketched in Fig. 1 where invariance is assumed along the y-axis. The background medium consists of two homogeneous non-magnetic (i.e., the magnetic permeability is everywhere the one of free-space μ_0) half-spaces separated by a planar interface at $z = 0$, their dielectric permittivity and wavenumbers being ε_u, k_u for the upper half-space and ε_l, k_l for the lower one. In particular, the lower medium is considered electromagnetically denser than the upper one, that is $k_l > k_u$.

The unknown targets are embedded in the lower half-space and assumed to reside within the segment SD (i.e., the investigation domain) located at $z_s < 0$. The incident field is radiated by a y-polarized line source of unitary amplitude. The time dependence is assumed equal to $e^{j\omega t}$ and omitted. The only y component of the scattered field is collected by

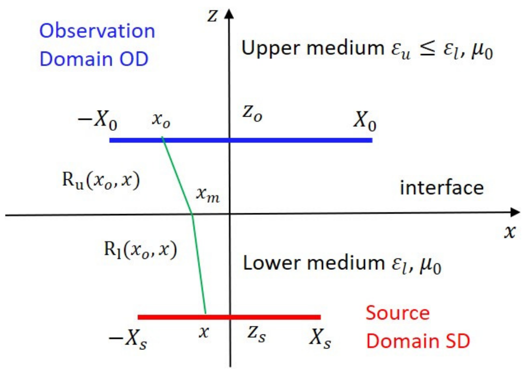

Figure 1. Geometry of the problem

exploiting a multi-monostatic measurement configuration, where the transmitting and receiving antennas are located at the same position in the upper half-space. The antenna system moves along the segment $OD = [-X_0, X_0]$(i.e., the observation domain) of the x-axis located at the height $z_o > 0$ and parallel to SD. Under the Born approximation, the contrast function χ and the scattered field are linked through the following scattering operator

$$\mathscr{A} : \chi \in L^2(SD) \rightarrow E \in L^2(OD) \tag{1}$$

where $L^2(SD)$ and $L^2(OD)$ represent the set of square integrable functions supported over SD and OD, respectively. The χ function is defined as $(\varepsilon_s(\dot{}) - \varepsilon_l)/\varepsilon_l$, with $\varepsilon_s(\dot{})$ being the dielectric permittivity of the unknown scatterer. The explicit form of the operator is given by

$$E(x_o) = k_l^2 \int_{SD} G^2(x_o,x)\chi(x)dx \quad x_o \in OD \tag{2}$$

with $G(\cdot)$ being the Green function pertinent to the two-layered background medium whose expression in terms of the Weyl expansion is

$$G(x_o,x) = \frac{i}{2\pi} \int_{-\infty}^{\infty} \hat{G}(k_x) e^{-jk_x(x_o-x)} dk_x \tag{3}$$

with

$$\hat{G}(k_x) = \frac{1}{2}\tau(k_x) e^{jk_{zl}z_s} e^{-jk_{zu}z_o}$$

$$\tau(k_x) = \frac{2}{k_{zl} + k_{zu}}$$

$$k_{zl} = \begin{cases} \sqrt{k_l^2 - k_x^2} & \text{if} \quad k_x \leq k_l \\ -j\sqrt{k_x^2 - k_l^2} & \text{if} \quad k_x > k_l \end{cases}$$

and

$$k_{zu} = \begin{cases} \sqrt{k_u^2 - k_x^2} & \text{if} \quad k_x \leq k_u \\ -j\sqrt{k_x^2 - k_u^2} & \text{if} \quad k_x > k_u \end{cases}$$

Note that the square in (2) takes into account the multi-monostatic measurement configuration. By considering $z_o, z_s > \lambda_l > \lambda_u$, the Green function in (3) can be approximated as [6]

$$G(x_o,x) \approx h(x_o,x) e^{-j\phi(x_o,x)} \tag{4}$$

where $h(x_o,x)$ takes into account the relevant amplitude factors, $\phi(x_o,x) = k_u(R_u + R_l)$, $R_u = \sqrt{(x_o - x_m(x_o,x))^2 + z_o^2}$, $R_l = n\sqrt{(x_m(x_o,x) - x)^2 + z_s^2}$, $n = \sqrt{\varepsilon_l/\varepsilon_u}$ being the refractive index, and $x_m(x_o,x)$ is the refraction point at the half-space interface according to Snell's law

$$\frac{x_o - x_m}{\sqrt{(x_o - x_m)^2 + z_o^2}} = n \frac{x_m - x}{\sqrt{(x_m - x)^2 + z_s^2}} \tag{5}$$

Let be $\{u_n, \sigma_n, v_n\}_{n=0}^{\infty}$ the singular system of $\mathscr{A}$, where σ_n are the singular values and u_n and v_n the singular functions. Consider the operator $\mathscr{A}\mathscr{A}^\dagger$, with $\mathscr{A}^\dagger$ being the adjoint operator of $\mathscr{A}$, whose eigenvalues are σ_n^2

$$\mathscr{A}\mathscr{A}^\dagger() = \int_{-X_o}^{X_o} K(x_o,x_o')()dx_o \tag{6}$$

where $K(x_o,x_o') = \int_{-X_s}^{X_s} H(x,x_o',x_o) e^{2j[\phi(x_o,x) - \phi(x_o',x)]} dx$ and $H(x,x_o',x_o) = k_l^4 h^2(x_o',x) h^{*2}(x_o,x)$.

Inspired by the approach developed in [13], it is possible to introduce the following variables

$$\xi(x_o) = \phi(x_o, -X_s) - \phi(x_o, X_s) \tag{7}$$

$$\gamma(x_o) = \phi(x_o, -X_s) + \phi(x_o, X_s) \tag{8}$$

Numerical results show that the kernel of the operator (6) $K(x_o,x_o')$ can be expressed as $e^{j[\gamma(x_o) - \gamma(x_o')]} K_b(\xi(x_o), \xi(x_o'))$ where $K_b(\xi(x_o), \xi(x_o'))$ can be approximated by a band-limited function with respect to the variable ξ whose band is 2. By exploiting the approach developed in [12], it can be proved that the eigenvalues of the continuous operator in (6) are equivalent to those of the following discrete eigenvalue problem for the matrix $\mathbf{A}$

$$\sigma_n^2 \mathbf{w_n} = \mathbf{A}\mathbf{w_n} \tag{9}$$

with $\mathbf{A}$ a matrix whose entries are

$$A_{ml} = \int_{-X_0}^{X_0} e^{-j[\gamma(x_{om}) - \gamma(x_o')]} K(x_{om},x_o') sinc(\xi(x_o') - \frac{l\pi}{\nu})) dx_o' \tag{10}$$

where

$$\xi(x_{om}) = \frac{m\pi}{\nu} \tag{11}$$

Here, $\nu \geq 1$ is the oversampling factor and $\mathbf{w_n}$ is an infinite dimensional vector whose m-th element is $v_n(x_{om})$. The discrete and continuous problems share the same eigenvalues. Note that (11) suggests to sample the observation domain with an uniform step in ξ variable equal to $\frac{\pi}{\nu}$. Moreover, since a non-linear relationship links ξ and x_o, the uniform sampling in ξ maps into a non-uniform sampling in x_o. This happens also for a homogeneous background medium [13]. Indeed, also there it is possible to introduce a similar transformation of the observation variable. However, here, the dependence of the transformation on x_o is different; this results in a different location of the sampling points.

From a theoretical point of view, $\mathbf{A}$ should have infinite dimension, however since its element are significant only for those indexes associated to samples falling within OD, a truncated version can be considered. Let be $\mathbf{A_N}$ a truncated version of $\mathbf{A}$, such as its elements are equal to A_{ml} for m and l lower than $N = \frac{\Delta\xi\nu}{\pi} + 1$, with $\Delta\xi = \xi(X_0) - \xi(-X_0)$. It is worth remarking that when ν is equal to 1, N coincides with the number of degree of freedom (NDF) equal to

$$NDF = \frac{\Delta\xi}{\pi} + 1 \tag{12}$$

that is the same number derived in [15]. Accordingly, in order to approximated the more significant singular values of $\mathscr{A}$, it is sufficient to fix $\nu > 1$.

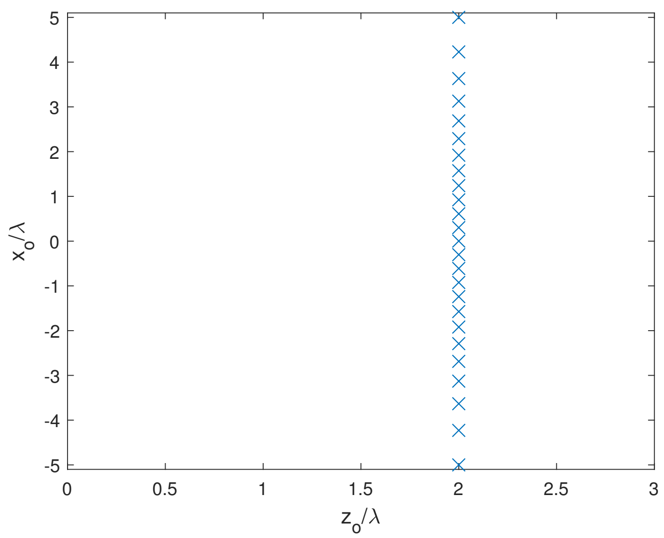

Figure 2. Sampling points x_{om} obtained by sampling ξ with a uniform step π/ν and $\nu = 1.15$.The configuration parameters are $a = 3\lambda$, $X_0 = 5\lambda$, $n = 3$, $z_o = 2\lambda$ and $z_s = -5\lambda$.

Fig. 2 shows the sampling points of the observation domain x_{om} when ξ is sampled uniformly with a step $\frac{\pi}{\nu}$ and $\nu = 1.15$. The configuration parameters are $a = 3\lambda$, $X_0 = 5\lambda$, $z_o = 2\lambda$ and $z_s = -5\lambda$.

In Fig. 3, the comparison between the singular values of the operator $\mathscr{A}$ (blue symbols 'o') and those of $\mathbf{A_N}$ (red symbols 'x') is shown. The matrix $\mathbf{A_N}$ is evaluated by sampling the observation domain as in Fig. 2 with $N = 25$. It is evident that the singular values before the knee, that is, in correspondence of $NDF = 20$, are very well approximated.

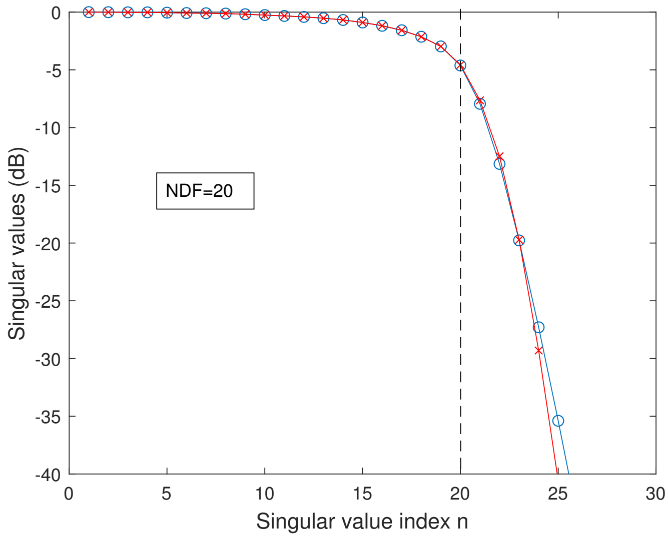

Figure 3. Comparison between the eigenvalues of the operator $\mathscr{A}\mathscr{A}^\dagger$ (blue symbols 'o') and those of $\mathbf{A_N}$ (red symbols 'x'). The matrix $\mathbf{A_N}$ is evaluated by sampling the observation domain as in Fig. 2 with $N = 25$. The configuration parameters are $a = 3\lambda$, $X_0 = 5\lambda$, $n = 3$, $z_o = 2\lambda$ and $z_s = -5\lambda$.

3 Conclusion

In this paper a linear inverse scattering problem for an inhomogeneous two-layered background medium has been addressed. In particular, in order to keep the notation simple, a scalar two dimensional configuration has been considered, where the investigation and the observation domains were parallel bounded strips separated by a distance greater than the wavelength. In particular, a multi-monostatic measurement configuration has been exploited. The focus, here, was on the determination of the positions where to collect the scattered field data. The developed strategy proposes to sample the scattered field in order to leave unchanged the more significant singular values of the continuous scattering operator. By doing so, it has been shown that the field must be sampled in a non-uniform way with respect the observation variable x_o.

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