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Cloaking Analysis by Accelerated-Particle-Swarm-Optimization Techniques

Alkmini Michaloglou[(1)] and Nikolaos L. Tsitsas[(1)]
(1) School of Informatics, Aristotle University of Thessaloniki, 54124 Thessaloniki, Greece

Abstract

Cloaking behavior of spherically-layered media is analyzed by means of a chaotic accelerated particle-swarm-optimization (chaotic APSO) algorithm. The optimization problem concerns the determination of the radii, the permittivities, and the permeabilities of a small number of magneto-dielectric layers covering a PEC core. The results of some preliminary numerical experiments are presented.

1 Introduction

A chaotic accelerated-particle-swarm-optimization (CAPSO) algorithm incorporating two types of chaotic maps is developed and applied for the analysis of the cloaking performance of a layered medium. Precisely, the considered optimization problem concerns the reduction in the scattering cross section of a spherical medium containing a perfect electric conducting (PEC) core covered by a number of magneto-dielectric layers. The medium is excited by an external magnetic dipole. The optimization variables are the radii, the permittivities, and the permeabilities of the covering layers. Some preliminary numerical results are presented showing that the bistatic scattering cross section is significantly reduced for a wide range of the observation angles for a coating composed by three layers with determined parameters.

Cloaking mechanisms of spherically-layered media and aspects of related optimization problems have been investigated by different methodologies; indicatively, we refer to [1]-[4]. In this work, we are focusing on media with a small number of covering layers composed of realizable magneto-dielectric materials. Optimizations for reduced scattered far-fields for the examined problem of dipole-excitation of a spherically-layered medium were performed in [5] by means of a standard particle-swarm-optimization algorithm. The present CAPSO algorithm is shown to provide more efficient optimization schemes.

2 Optimization Problem

A layered spherical medium V with radius a_1 is considered, the interior of which is divided by $P-1$ concentric spherical interfaces $r=a_p$ $(p=2,\dots,P)$ into $P-1$ homogeneous dielectric layers V_p $(p=1,\dots,P-1)$, consisting of materials with dielectric permittivities ε_p and magnetic permeabilities μ_p, and surrounding a PEC core (layer V_P). The exterior V_0 of V is characterized by permittivity ε_0, permeability μ_0, and wavenumber k_0. The medium V is excited by an external magnetic dipole, with position vector $\mathbf{r}_0$ on the z-axis and with dipole moment along the direction $\hat{\mathbf{y}}$.

The exact solution of the scattering problem is determined analytically in [11] and [12], yielding the following expression of the total scattering cross section

$$\begin{aligned}\sigma^{\mathrm{t}}(\mathbf{r}_0) &= \frac{1}{4\pi}\int_{S^2}\sigma(\theta,\phi;\mathbf{r}_0)\mathrm{d}s(\hat{\mathbf{r}})\\ &= \frac{2\pi}{k_0^2}\sum_{n=1}^{\infty}(2n+1)\left[|\gamma_n|^2+|\delta_n|^2\right], \qquad (1)\end{aligned}$$

where $\sigma(\theta,\phi;\mathbf{r}_0)$ is the bistatic (differential) scattering cross section and γ_n and δ_n are determined coefficients, while S^2 denotes the unit sphere in $\mathbb{R}^3$.

3 Chaos-enhanced Accelerated Particle Swarm Optimization

The original particle swarm optimization (PSO) algorithm was introduced by Kennedy and Eberhart in 1995 [6] and is now a well-established optimization algorithm that is applied in many variations and standards. It is based on the behavior and movement of swarms, like e.g. flocks of birds, in nature. Each particle of the swarm is attracted by a global best $\mathbf{g}^*$ (the best result achieved in the whole swarm until the present moment) and its own best location $\mathbf{x}^*$ in history. Furthermore, for each particle, two characteristics are maintained describing its movement: position $\mathbf{x}$ and velocity $\mathbf{u}$. Additional parameters may include randomness, inertia, and learning rates and depend on the type of the applied PSO algorithm [7, 8].

In 2008, Yang proposed the *Accelerated Particle Swarm Optimization* (APSO) algorithm, which follows a different perspective on the description of the particles [9]. Specifically, the goal is to simplify the particles' movement without sacrificing the PSO algorithm's efficiency. So, if it is considered that the PSO's individual best $\mathbf{x}^*$ is used as a means of diversity, then it can be simulated by randomness. As a result, in the APSO algorithm, the velocity's $\mathbf{u}$ update formula is deemed unnecessary since it can be included in the position's $\mathbf{x}$ update formula [9], which is described by the following simple step:

$$\mathbf{x_i}^{t+1} = (1-\beta)\mathbf{x_i}^{t} + \beta \mathbf{g}^{*} + \alpha r. \tag{2}$$

Usually, the initial values of the parameters α and β lie in the intervals $[0.1, 0.4]$ and $[0.1, 0.7]$, respectively. However, it is noted that they should generally be related to the scales of the problem's parameters. Precisely, the code examples in [9] propose a formula that initializes and upgrades α in accordance to both a value set by the programmer and the set number of iterations. It is also considered as an improvement to gradually reduce randomness, so that α gets updated through a monotonically decreasing function in each iteration.

The *Chaos-enhanced APSO* (also called as the *Chaotic APSO: CAPSO*) is another variation of the APSO designed to offer improved characteristics [10]. For this algorithm, it is stated that the parameter β has no practical reason of remaining constant. Specifically, if it is well-tuned, it can lead to acceleration of the algorithm's convergence. The methodology suggested for updating the parameter β is that of the *Chaotic Maps*, which, also, improve the ability of the APSO to escape local optima in a multimodal landscape.

Various chaotic maps have been proposed; see e.g. [10]. Besides, it is noted that they should be normalized in order for the values to be in the range of $[0, 1]$. In this work, we employ two of them, the *sinusoidal* map and the *singer* map, which, according to [10], have been the most successful in testing. These maps are defined as follows:

Sinusoidal Map:

$$x_{k+1} = a x_k{}^2 \sin(\pi x_k) \tag{3}$$

Singer Map:

$$x_{k+1} = \mu(7.86x_k - 23.31x_k{}^2 + 28.75x_k{}^3 - 13.302875x_k{}^4), \tag{4}$$

where $\mu \in [0.9, 1.08]$.

The CAPSO algorithm is presented below. Parameter α is updated through a monotonically decreasing function. Convergence can be checked in multiple ways, usually standard deviation is chosen in order to ascertain that the particles move towards the same solution.

Algorithm 1: Chaos-enhanced APSO (Chaotic APSO)

Input: l, u, it_{max}, n
Output: A swarm S of n particles and their final positions
Initialize S of size n with random positions wrt the upper and lower bounds u and l;
Initialize parameters α and β;
Initialize iteration, it = 1;
Set convergence as $false$;
while $it \leq it_{max}$ *AND convergence* $== false$ **do**
    Find global best g^* for iteration it;
    Update α;
    Update β through a chaotic map;
    for *Each particle in S* **do**
        Update locations using Equation (2);
    end
    Update the iteration counter, $it = it + 1$;
    Update convergence;
end

4 Numerical Results

As the objective function of the optimization problem, we consider the normalized total scattering cross section $\sigma^{\mathrm{t}}(\mathbf{r}_0)/(\pi a_{\mathrm{PEC}}^2)$, where a_{PEC} is the radius of the PEC sphere to be cloaked, which was chosen constant at $k_0 a_P = 2\pi$ (one free-space wavelength). The differences $k_0(a_{p+1} - a_p)$ between two consecutive layers radii were considered in the range $[\frac{\pi}{10}, \frac{\pi}{2}]$. The values of ε_p and μ_p were allowed to vary in the range [0.5,5]. The distance r_0 of the dipole from the scatterer was taken $r_0 = 10a_1$, for which case the obtained far-field results are close to the ones corresponding to plane-wave incidence [11].

We applied the developed chaotic APSO algorithm for a spherical medium with $P = 4$ total number of layers. To demonstrate the actual reduction in the far-field with respect to the angles of observation, we depict in Fig. 1 a representative plot of the normalized bistatic scattering cross section $\sigma(\theta, \phi; \mathbf{r}_0)/(\pi a_{\mathrm{PEC}}^2)$ as function of the angle θ in the xOz and yOz planes. Significantly reduced far-field contributions with respect to the bare PEC sphere are observed for a wide range of the observation angles.

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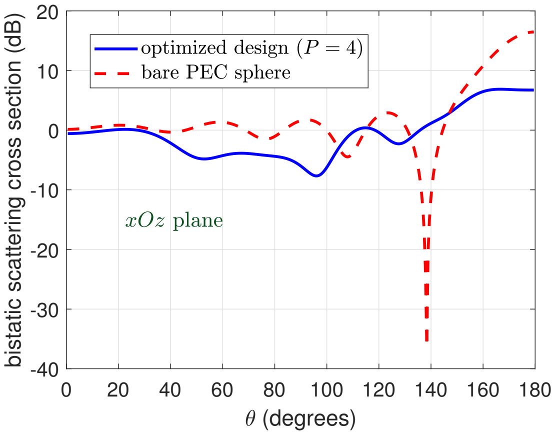
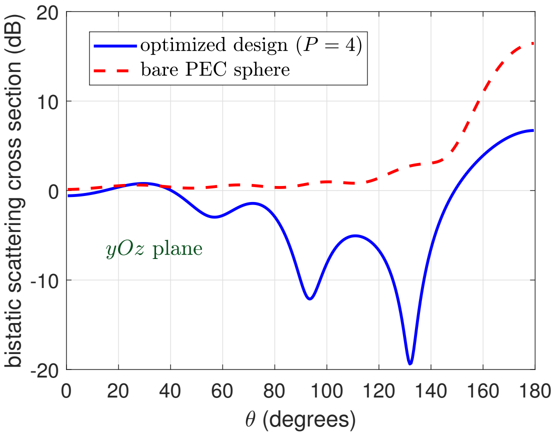


(a)



(b)

Figure 1. Normalized bistatic scattering cross sections versus the observation angle θ for $P = 4$ optimized layers with parameters computed by the CAPSO algorithm; (a) and (b) refer to xOz and the yOz planes, respectively.

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