An Asymptotic Theory of the Solar Glory

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The phenomenon of the solar glory consists in color rings surrounding the shadow cast by an observer on a layer of water droplets as in clouds, mist and fog. In contrast to other similar atmospheric phenomena, like rainbows or coronas, and despite multiple attempts over more than half of a century (most notably by van de Hulst and Nussenzveig), a satisfactory theory of the solar glory has not been derived until recently. An asymptotic study of backscattering from an electrically large low-absorption sphere [1] has suggested that the glory might be caused by the whispering-gallery modes propagating in the interior of water droplets. A modification of the Watson complex integration method outlined in [2] has resulted in a complete asymptotic theory of the phenomenon. The main result of the theory is the following expressions for the electric and magnetic scattering coefficients:

$$S_{e,m}(\theta) = S_{e,m}^{\mathrm{ref}}(\theta) + \sum_{p_{e,m}=1}^{P_{e,m}} \delta_{p_{e,m}}^{e,m} J_{-}\left(\nu_{p_{e,m}}^{e,m}\theta\right) + \sum_{p_{m,e}=1}^{P_{m,e}} \delta_{p_{m,e}}^{m,e} J_{+}\left(\nu_{p_{m,e}}^{m,e}\theta\right),$$

where θ is the bistatic angle ($\theta = 0$ in the backscattering direction) and $J_{\pm}(w) = J_0(w) \pm J_2(w)$ with $J_{0,2}(w)$ being the Bessel functions. The terms S_e^{ref} and S_m^{ref} describe the rays reflected from the exterior of the sphere. The sums are the residue contributions from the electric and magnetic poles $\nu_{p_{e,m}}^{e,m}$ of the so-called *horizontal* branch, which, apart from small imaginary parts, fill the interval $0 < \nu < k_1 a$ on the complex ν plane. Here, a is the radius and k_1 is the wavenumber in the material of the sphere. Every residue term describes a kind of a whispering-gallery mode propagating on the concave side of the spherical boundary. The number of terms $P_{e,m}$ in the sums is finite, and the coefficients $\delta_{p_{e,m}}^{e,m}$ that correspond to the poles with $\nu_{p_{e,m}}^{e,m} \approx ka$ (k is the wavenumber in the air, $k < k_1$) are much larger than the rest coefficients (Fig. 1). This property permits the general conclusion that solar glories are mainly formed by several whispering-gallery modes excited near the equator, where the incidence direction is almost tangential to the sphere, including the case of lateral excitation ($ka < \nu_{p_{e,m}}^{e,m} < k_1 a$).

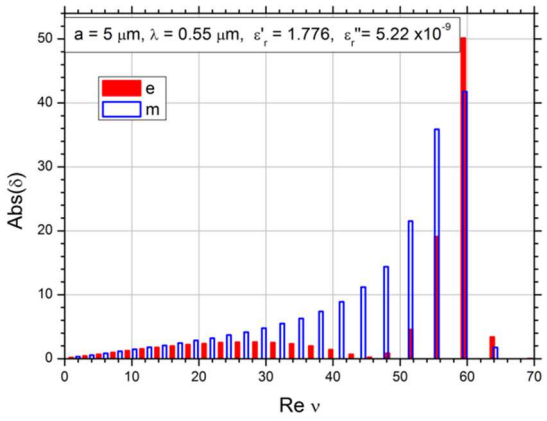

Figure 1. Amplitudes of $\delta_{p_{e,m}}^{e,m}$ for a water droplet of the size typical for mist, fog and clouds ($ka \approx 57$).

References

[1] A. V. Osipov and S. A. Tretyakov, *Modern Electromagnetic Scattering Theory with Applications*, Wiley, 2017.

[2] A. V. Osipov, "Towards a Rigorous Electromagnetic Theory of the Solar Glory," *URSI Commission B International Symposium on Electromagnetic Theory (EMTS 2019), May 27-31, 2019, San Diego, USA*.