Abstract

We study spontaneous coherent super-radiative undulator emission range from a short dense electron bunch in the Terahertz frequency. Spontaneous radiation takes place, if a bunch is shorter than the radiated wavelength. If the group velocity of the radiated wave is close to the bunch velocity, this is a process of the spontaneous radiation and then amplification of a single wave cycle. As a result, formation of an ultra-short (several cycles) wave packet occurs. Since the radiated field prevents the destruction of the bunch under the influence of its own Coulomb fields, the process of effective spontaneous coherent radiation is maintained for a relatively long length of the electron-wave interaction region. In the optimized system, this results in a high (10-15%) efficiency of the radiation process, as well as in a high intensity (several hundred MV/m) of the field accommodated in the short wave packet.

1 Introduction

Photo-injectors ensure formation of dense picosecond electron bunches with charges of the order of 1 nC and particles energy at the level of 3-7 MeV [1-3]. These parameters are attractive for creation of powerful sources of coherent THz radiation. In particular, it is attractive to use the coherent spontaneous undulator radiation from a short bunch. If the effective length of the electron bunch is shorter than the wavelength of the radiated wave, then this kind of emission starts in radiation section immediately (without the process of the induced electron bunching), as the wave packets emitted by each of the electrons add up basically in phase. Advantages of this spontaneous coherent emission regime (as compared to the regime of the induced emission traditionally used in electron masers) are short and simple microwave system (with no wave feedback system and seeding wave signal), possible high efficiency of the radiation process, and the fixed phase of the radiated wave packet [4-7].

However, an important problem is that the initial coherence of radiation is rapidly broken due to increase in the bunch length caused by the strong Coulomb repulsion of particles inside a bunch. A possible solution of this problem is stabilization of the bunch length (or even compression of the bunch in its axial direction) provided by the radiated wave field of the bunch (super-radiative self-compression [5]).

We describe a process of spontaneous coherent super-radiative undulator emission from a short dense electron bunch. A principal difference of this process as compared to the induced super-radiative emission process [8,9] is formation of an ultra-short wave packet during motion of the bunch through just several undulator periods. The spontaneous radiation process starts just from the first undulator period, where a quasi-unipolar wave pulse (just one cycle of the wave) is radiated. Since the group wave velocity coincides with the bunch velocity, in the next undulator periods we see amplification of this “first” wave cycle. However, the diffraction effects lead to a more complex shape of the wave packet. It is important that the radiated field prevents the destruction of the bunch under the influence of its own Coulomb fields. As a result, the process of spontaneous radiation continues for a long length, which ensures a high efficiency of this process and a high intensity of the field accommodated in a short wave packet.

2 Effect of Super-Radiative Self-Compression

Consider the spontaneous emission of a dense electron bunch moving in a waveguide placed in the periodic field of the undulator (Fig. 1), in the case when the longitudinal velocity of the particles is close to the group velocity of the emitted pulse (superradiation). In the superradiance regime, the electron-wave interaction is most effective, since in this case the bunch is located in the region of the maximum of the emitted pulse.

![Figure 1](image-url)

Figure 1. Spontaneous coherent super-radiative undulator emission from a short dense electron bunch. The bunch moving in the waveguide immersed in the periodical undulator field. At different undulator periods, the bunch radiates the waves packet in the same cycle.

At the beginning of the radiation process, the center of the electron bunch is located at the maximum of the
decelerating phase of the wave (see Fig. 2, a). Since the bunch is decelerated, the maximum decelerating phase of the wave is shifted to the left after the center of the bunch.

The bunch is slightly stretched in the longitudinal direction due to the Coulomb interaction (see Fig. 2, b), however, the particles moving in front are slowed down by the wave field, while the electrons flying in the tail, being in the stable neutral phase of the wave, are held by its field (see Fig. 2, c). Thus, the interaction of electrons with the wave leads to a reduction in the length of the bunch.

Consider the radiation of the bunch with an initial duration 1. ps \((L_e = 0.3 \text{ mm})\), approximately one-fourth part of the Doppler-upshift of an undulator period \(\Lambda_u = 100 \text{ mm}\), a transverse size \(2R_c = 4\text{mm}\), and the energy 5MeV in the undulator with undulator parameter \(K = 0.7\). The radiation occurs in the waveguide with a radius \(R_w = 3.25 \text{ mm}\) (to provide radiation in the regime, which is close to the group synchronism). The dynamics of the charge density is shown in the figure 3. Black curves accord to the case, when the bunch is stretched under the action of Coulomb fields only. Green fills accord to the case, when the bunch radiates in the spontaneous regime, which prevents increase in length caused by coulomb interaction. Initially the amplitude of the radiated wave field is small in comparison with Coulomb fields, but after several periods we can notice significant stabilization and even compression after approximately 10 periods (Fig. 3). After 12 periods a bunch length increase by factor 5 due to the Coulomb repulsion of particles, but the electron-wave interaction provides keeping of a main part of charge at the boundaries of the initial longitudinal size, in addition there is a peak with a double density value.

**Figure 2.** Simulation of the 1 THz third-harmonic gyrotron. The shape of the operating cavity, starting currents of the operating (red curve) and main parasitic (blue curve), and the output powers of these modes versus the time.

**Figure 3.** The dynamic of charge density in the wave and Coulomb fields (green fills) and in the case of only Coulomb fields (black curves) in various points of interaction.

### 3 Spontaneous Radiation

A short electron bunch moving in the undulator, during its first undulatory oscillation, emits one cycle of an electromagnetic wave, the length of which corresponds to the Doppler-upshift of the period of the undulator (Fig. 1). In this case, since the group velocity of the emitted wave packet coincides with the longitudinal velocity of electrons, the cycle of the wave in the first approximation, does not leave the e-bunch, and so when electron bunch passes the next period of the undulator this cycle is amplified. However, there is dispersion in a waveguide, namely after \(N_u\) period a duration of the radiated pulse increase to the length proportional to the number of passed periods devided by the longitudinal Lorentz-factor, namely \(N\Lambda_u/\left(2\gamma_{z,0}^2\right)\).
Figure 4. Efficiency of radiation as a function of axial coordinate normalized by the undulator period in the regular waveguide (blue dashed curve) and in the profiled waveguide (solid purple curve) as a function of axial coordinate normalized by the undulator period. The red curve describes profiling of the waveguide.

Figure 5. The radiated electric field (solid purple curves – electric field of the radiated wave pulse in the case of the profiled waveguide, dashed blue curves accord to the case of the regular waveguide) and charge distribution (green fill accords to the profiled waveguide, black curves to the regular waveguide) in different points of interaction.

We consider the radiation of the bunch with the same parameters (Sect. 2). The dashed blue curve in the Fig. 4 accords to the efficiency of radiation in this case. It is sufficiently high (5 %), but the radiated pulse spreads due to the dispersion in the waveguide, so the amplitude of the electric field of the radiated wave decreases. Profiling of the waveguide radius allows to improve the efficiency and the amplitude of the radiated field. A decrease in the radius leads to an increase in the phase velocity of the radiated wave, and vice versa. Starting from a radius, which slightly exceeds the value according to the group synchronism, it is possible to capture a greater part of particles, decreasing in radius leads to decrease in energy of the trapped particles, starting to increase radius again, it is possible to increase a number of the trapped particles. Compare the results for a regular waveguide and for a profiled waveguide are shown in the Figs. 4 and 5. Results of numerical simulation show, that a smooth decrease in the waveguide radius from 3.9 mm to 3.2 and following increasing to 3.9 mm significantly improves conditions for radiation.

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5 References