



## A Tutorial on Electromagnetic Inverse Scattering and Inverse Source Problems

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This Tutorial is concerned with some of the fundamentals of electromagnetic inverse problems. The target audience for this Tutorial are those who are to begin their research in the area of electromagnetic inverse problems and would like to learn some of its basic considerations and fundamentals.<sup>1</sup> Electromagnetic inverse problems deal with determining some properties of an investigation domain from its associated external electromagnetic data. The process of inferring (or, reconstructing) the properties of the investigation domain from its external electromagnetic data is referred to as electromagnetic inversion. In other words, electromagnetic inversion processes exterior electromagnetic effects to reconstruct their original cause. The computational aspect of this processing often involves different steps such as modelling, optimization, regularization, etc.

Broadly speaking, electromagnetic inversion has two main application categories: (1) characterization, imaging, and remote sensing, as well as (2) design. The first category deals with applications such as microwave biomedical imaging, antenna characterization, geophysical remote sensing, etc. On the other hand, the second category is concerned with design applications, and may be for example applied to metasurface design, lens design, electromagnetic cloak design, etc. The main difference between these two application categories lies in the type of electromagnetic data to be inverted by the utilized electromagnetic inversion algorithm. In the first category (characterization, imaging, and remote sensing), the electromagnetic data are a set of measured data collected from the investigation domain. For example, in microwave breast imaging, the electromagnetic data are often the scattered fields emanating from the illuminated breast. These measured scattered fields are then inverted to reconstruct the unknown relative complex permittivity profile of the breast. However, in the second category (design), the electromagnetic data are a set of desired specifications to be achieved, e.g., desired power patterns or desired performance criteria such as antennas' main beam directions and half-power beamwidths. In design applications, the equivalent currents (or, dielectric profile, magnetic profile, etc) are often to be found such that the desired specifications can be met as much as possible. In addition to this wide range of applications, the electromagnetic inversion framework is applicable to a wide range of frequencies; however, the focus of this Tutorial is on the microwave frequency range.

In electromagnetic inversion, the investigation domain may include scatterers such as in vivo biological tissues, snow-covered sea ice, or a dielectric lens, which are irradiated by some incident electromagnetic fields. For these cases, the goal is to characterize these scatterers, e.g., in terms of their relative complex permittivity and/or permeability profiles, by processing their associated external electromagnetic response. For these inversion scenarios, the electromagnetic inverse problem is referred to as the electromagnetic inverse scattering problem. On the other hand, the investigation domain may include electromagnetic sources such as antennas or unintended radiators. In such cases, the purpose of electromagnetic inversion is often to characterize these sources in terms of their equivalent electric and/or magnetic currents from their near-field or far-field measured or desired data. For example, in antenna diagnostics, the electromagnetic inversion framework can be used to reconstruct the equivalent currents of the antenna under test from near-field measurements. This category of inverse problems is referred to as electromagnetic inverse source problems.

After a brief overview of different application areas for electromagnetic inversion, this Tutorial begins with describing the relationship between electromagnetic inverse source and inverse scattering problems. One of the common challenges associated with solving electromagnetic inverse problems is then considered: how to treat the ill-posedness of the associated mathematical problem via appropriate regularization techniques. We then focus on two specific application areas: (i) microwave imaging and (ii) macroscopic metasurface design. Some important aspects associated with using the electromagnetic inversion framework for these two application areas, e.g., how to form the cost functional to be optimized, how to handle the nonlinearity of the problem, how to calibrate the data, etc, are then discussed. Throughout this Tutorial, it is emphasized that electromagnetic inversion provides a flexible and systematic framework that can deal with different forms of electromagnetic data.

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<sup>1</sup> This URSI Early Career Representative (ECR) Tutorial is based on a similar Tutorial presented by the author at the 32nd URSI General Assembly and Scientific Symposium, 2017, and later at the 2nd URSI Atlantic Radio Science Meeting (AT-RASC), 2018. Most of the materials covered in this Tutorial are based on previously published works whose citations will be provided at the Tutorial presentation.