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On the Inclusion of Thin Sheets in the Global Multi-trace Method

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Abstract

In this paper, a global multi-trace method for the scattering of time-harmonic waves by a structure that can contain, in addition to dielectric and perfectly conducting regions also perfectly conducting thin sheets, is presented. The method is direct in the sense that the unknowns are traces and jumps of the fields in the structure. The flexibility of the method and the correctness of the solution will be demonstrated by realistic examples.

1 Introduction

Electromagnetic scattering involving composite objects made up of several layers of metallic and penetrable materials are time and again encountered in practical engineering problems. For example, in electronics, aerospace and defence, thin metallic sheets (so-called 'shielding sheets'), or coatings are often used to provide strong protection for electronic devices from undesirable electromagnetic interference[1]. Modelling geometries of this nature however poses a challenge as special care and attention must be paid to the electromagnetic traces at the junctions where two or more materials meet.

Domain decomposition boundary element methods (DD-BEM) provides an appealing approach for dealing with such scattering problem involving multiple media or domains. Here the composite structure is partitioned into several sub-domains and the interaction between individual domains are enforced using transmission conditions. The main reasons to consider domain decomposition approaches are (i) the opportunities for effective preconditioning, and (ii) the ability to undertake computation in parallel to speed up the global solution.

Of particular interest is the Multi-trace formulation (MTF). Multi-trace formulations provide a clean domain decomposition approach that is amenable to fast solution techniques based on Calderon preconditioning [2]. In [3], a MTF approach that can be used for the solution of scattering/transmission problems involving both penetrable and impenetrable domains is introduced and analysed. The authors offer both direct and indirect Combined Field Integral Equation (CFIE)-type formulations that are not susceptible to interior resonances, regardless the details of the geometric subdivision.

In this contribution, a MTF formulation is introduced that can model scattering/transmission problems involving systems that contain thin sheets. In [4], an indirect method based on only electric currents (the so-called ECF) is presented and can deal with perfectly conducting regions with non-zero volume. The method presented here is a direct method. The unknowns can be readily interpreted as field values (or jumps of field values), without additional post-processing. The method here is a direct extension of the Poggio-Miller-Chan-Harrington-Wu-Tsai (PMCHWT) method for single domain transmission problems and shares with that method that it is not susceptible to interior resonances. The formulation is introduced, a discretisation strategy is put forward, and the implementation challenges are discussed.

2 Formulation

This contribution is concerned with time-harmonic transmission problems at fixed frequency ω. Consider a device that is partitioned in a number of domains $\Omega_0,\ldots,\Omega_S$, where Ω_0 is the only unbounded domain. The normal to $\Omega_s, s=1,\ldots,S$ is denoted $\hat{\boldsymbol{n}}_s$ and is pointed outward of Ω_s. Each domain is occupied by a material characterized by permittivity and permeability (ε_s,μ_s), or equivalently by the wave number and impedance (κ_s,η_s).

In addition, it is assumed that the geometry contains a number of PEC sheets Γ_r, $r=S+1,\ldots,S+R$. Each sheet radiates into a single domain. For the common case where sheets are deposited as coatings on neighboring penetrable regions, multiple choices for this assignment procedure are possible. The arbitrarily but consistently chosen normal fields to Γ_r are denoted $\hat{\boldsymbol{n}}_r$.

The essence of the method introduced here can be clarified by considering the special case of a single penetrable region Ω_1, and a single sheet Γ_2 that radiates in the unbounded region $\Omega_0=\mathbb{R}^3\setminus\Omega_1$. The sheet is geometrically embedded in $\partial\Omega_0\cap\Omega_1$, i.e. it is a coating applied onto the penetrable region (Fig. 1).

For a field $(\boldsymbol{e}, \boldsymbol{h})$ in Ω_0 to be an eligible solution to the scattering problem, the tangential electric and magnetic field just outside of $\partial\Omega_s$ should vanish, as should the (average) tangential electric field at $\Gamma_r^{(s)}$. Taking into account both the equivalent currents on $\partial\Omega_1$ and the induced current on the sheet Γ_3 radiate into Ω_0 leads to the following constraints on the field traces and jumps:

$$0 = \begin{pmatrix} \boldsymbol{e} \times \hat{\boldsymbol{n}}_1 \\ \hat{\boldsymbol{n}} \times \boldsymbol{h}_1 \\ \{\boldsymbol{e} \times \hat{\boldsymbol{n}}_2\} \end{pmatrix} = \begin{pmatrix} K - \frac{1}{2} & -T & -T \\ T & K - \frac{1}{2} & K - \frac{1}{2} \\ K - \frac{1}{2} & -T & -T \end{pmatrix} \begin{pmatrix} \boldsymbol{m}_1 \\ \boldsymbol{j}_1 \\ \boldsymbol{j}_2 \end{pmatrix} + \begin{pmatrix} \boldsymbol{e}^i \times \hat{\boldsymbol{n}}_1 \\ \hat{\boldsymbol{n}}_1 \times \boldsymbol{h}^i \\ \boldsymbol{e}^i \times \hat{\boldsymbol{n}}_2 \end{pmatrix}, \quad (1)$$

with $\boldsymbol{m}_1 = \boldsymbol{e} \times \hat{\boldsymbol{n}}_1, \boldsymbol{j}_1 = \hat{\boldsymbol{n}}_1 \times \boldsymbol{h}, \boldsymbol{j}_2 = [\hat{\boldsymbol{n}}_2 \times \boldsymbol{h}]$, and where square brackets denote the jump across the sheet. The off-diagonal contributions $1/2$ should be interpreted as:

$$\frac{1}{2} j := \frac{1}{2} \int_{\Gamma_q} \delta(x - y) j(y) dy \quad x \in \Gamma_p. \quad (2)$$

They only contribute at $\Gamma_p \cap \Gamma_q$. Unfortunately their assembly cannot be performed by classic finite element method assembly routines. Correct assembly requires the ability to look up geometrically coinciding elements from the two interacting surfaces Γ_p and Γ_q. The sign of these off-diagonal contributions depends on the choice of normal field on the sheets.

For the same field to be a solution of a solution in Ω_1, it needs to fulfill:

$$0 = \begin{pmatrix} -K' - \frac{1}{2} & T' & 0 \\ -T' & -K' - \frac{1}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{m}_1 \\ \boldsymbol{j}_1 \\ \boldsymbol{j}_2 \end{pmatrix}. \quad (3)$$

In the spirit of the PMCHWT equation, the outer equation and the inner equation are subtracted, yielding:

$$0 = \begin{pmatrix} K + K' & -T - T' & -T \\ T + T' & K + K' & K - \frac{1}{2} \\ K - \frac{1}{2} & -T & -T \end{pmatrix} \begin{pmatrix} \boldsymbol{m}_1 \\ \boldsymbol{j}_1 \\ \boldsymbol{j}_2 \end{pmatrix} + \begin{pmatrix} \boldsymbol{e}^i \times \hat{\boldsymbol{n}}_1 \\ \hat{\boldsymbol{n}}_1 \times \boldsymbol{h}^i \\ \boldsymbol{e}^i \times \hat{\boldsymbol{n}}_2 \end{pmatrix} \quad (4)$$

Finally, to bring out the symmetry, the first and second row are interchanged and the sign of the single layer contributions located on the diagonal are brought in agreement:

$$0 = \begin{pmatrix} T + T' & K + K' & K - \frac{1}{2} \\ -K - K' & T + T' & +T \\ -K + \frac{1}{2} & +T & +T \end{pmatrix} \begin{pmatrix} \boldsymbol{m}_1 \\ \boldsymbol{j}_1 \\ \boldsymbol{j}_2 \end{pmatrix} + \begin{pmatrix} \hat{\boldsymbol{n}}_1 \times \boldsymbol{h}^i \\ -\boldsymbol{e}^i \times \hat{\boldsymbol{n}}_1 \\ -\boldsymbol{e}^i \times \hat{\boldsymbol{n}}_2 \end{pmatrix} \quad (5)$$

Note that the unknowns are defined on entire domain boundaries or entire sheets. For instance, the vanishing of the electric trace on that portion of $\partial\Omega_1$ that is covered by the metallic coating does not need to be implemented as an essential condition i.e. the space of candidate solution does not need to be limited to only those fields that already exhibit this property. Instead, the PEC condition at the sheet emerges as a natural condition, i.e. it is one of the distinguishing properties of the solution of the system. Another advantage, and the main motivation of the originators of multi-trace integral equations, is that the system is amenable to Calderon preconditioning.

3 Numerical Results

We first consider the case of scattering by a single domain Ω_1 partially coated by a thin PEC sheet $\Gamma_1^{(2)}$ as illustrated in figure 1. Ω_1 has wave number $\kappa_1 = 2.4\kappa_0$ and impedance $\eta_1 = \eta_0$ and is surrounded by an unbounded medium Ω_0 with material properties $\kappa_0 = 3.0$, $\eta_0 = 1.0$. The structure is

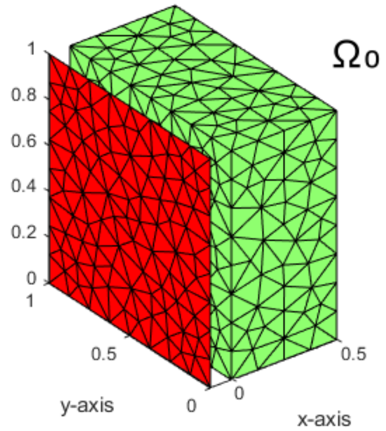

Figure 1. Rectangular block with PEC sheet (colored:red), and dielectric (colored:green) approximated by a triangular mesh comprising 712 facets. The gap between the layers is added for clarity and to illustrate the '*gap idea*' in the MTF formulation, and is not included in the simulation of the actual geometry.

illuminated by a plane wave $\boldsymbol{e}^i = \hat{x}e^{-j\kappa_0 z}$, $\boldsymbol{h}^i = -\frac{1}{j\kappa_0\eta}(\nabla \times \boldsymbol{e}^i)$.

The unknown fields are approximated as linear combinations of Rao-Wilton-Glisson (RWG) functions. The equations are tested by the set of *rotated* $n\times$ RWG functions. The number of degrees of freedom on Γ_1 is 1068, whereas the number of DoFs on Γ_2 is 260.

Figures 2 (left and middle) show a colormap of the near-field in the system. Fig 2 (middle) reveals that the h_y-field is continuous across the dielectric portion $\partial\Omega_1$ of the structure while there is a discontinuity across the PEC interface. This jump corresponds to the induced electric currents on the surface of the PEC. This is expected and is in accordance with boundary conditions. Fig. 2 (right) shows the z

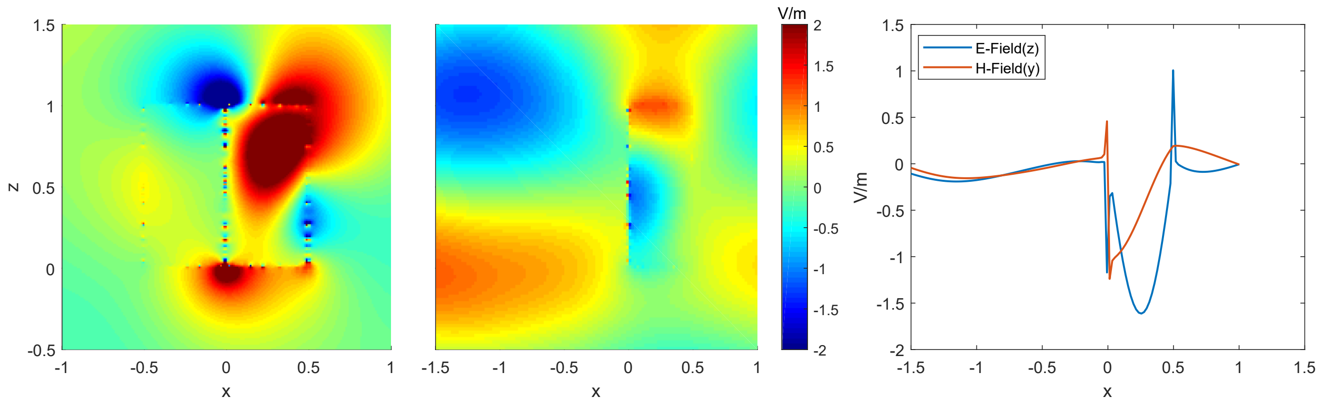

Figure 2. *Left*: Colormap of the z component of the electric near field E_z (V/m) plotted along the x and z axes at $y = 0.5$. *Middle*: Colormap of the y-component of the magnetic near field H_y (V/m) plotted along the x and z axes at $y = 0.5$. *Right*: Traversing the structure along the x-axis, the z-component of the electric field can be seen to be continuous and tend to zero as the sheet is approached. The y-component of the magnetic field is continuous, except at the sheet, where it jumps proportionate to the induced current.

and y components of the total electric and magnetic fields respectively, as the cuboid is traversed at the center from left to right on the x-axis. Field continuity is as we expected and the electric field vanishes at the PEC sheet as required.

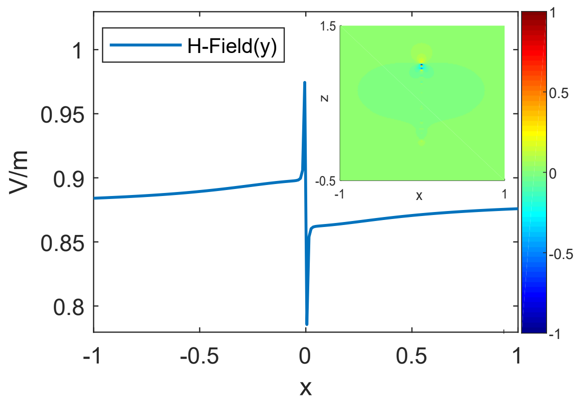

Figure 3. For scattering by an object made of the same medium as the background domain, the y-component of the total magnetic field is continuous and equal to the incident field, except at the sheet. The total E_z field is zero (inset).

For verification purposes, the case where the medium occupied by Ω_1 is chosen to be the same as the enveloping domain Ω_0 is considered. Here it is expected that Ω_1 is transparent and the solution is simply equal to the incident field. As demonstrated in Fig. 3 the total magnetic field just inside and outside Ω_1 remains the same, continuous, and is roughly equal to the incident magnetic field. A slight discontinuity across the thin PEC sheet can also be observed. Furthermore, the tangential electric field is also continuous and tends to zero at both sides of the PEC (see Fig. 3 inset).

To demonstrate the flexibility and robustness of the approach introduced here, consider the geometry from Fig. 4.

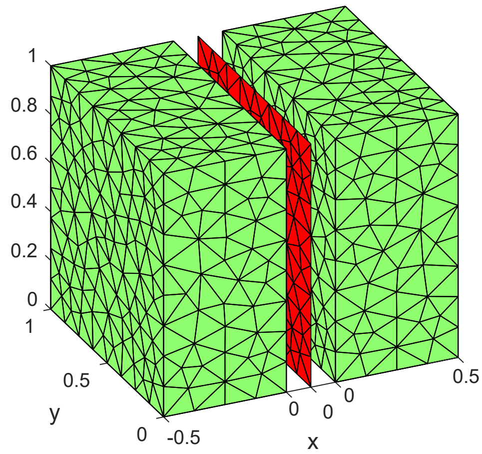

Figure 4. Composite structure featuring two dielectric layers (colored:green) and a thin PEC sheet (colored:red).

A thin sheet is sandwiched in between two dielectric layers. In this setup, not two but three geometrically coinciding surfaces interact through the external domain. To correctly implement this scenario, special care needs to be taken in deducing the signs of the off-diagonal $1/2$ contributions. Indeed, upon arbitrarily assigning an orientation and accompanying field of normals $\hat{\boldsymbol{n}}_3$ to the thin sheet, this normal field will point *towards* one of the penetrable domains, and *away* from the other. Because the system contains an additional penetrable region, a new pair of traces $(\boldsymbol{m}_3, \boldsymbol{j}_3)$ appears in the MTF formulation.

The second dielectric layer Ω_3 has material properties $\kappa = 1.4\kappa_0$ and $\eta = \eta_0$, and the combined structure is illuminated with the plane wave. Figures 5(a) and (b) shows the induced surface electric currents on all domains. Fig 5(b) highlights the significant current densities on the surface of the PEC sheet with higher current magnitudes along the

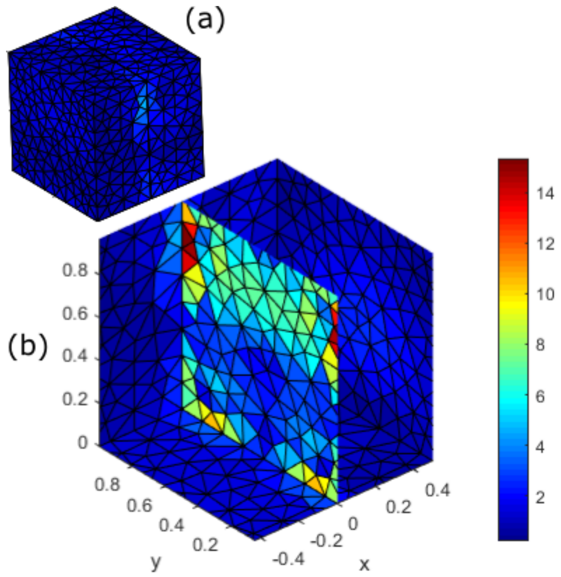

Figure 5. Surface electric currents (in A/m) induced on the surface of the dielectric and PEC layers.

boundaries. This is due to a singular behaviour of the current at the edges typically expected in the simulation of thin conductive sheets.

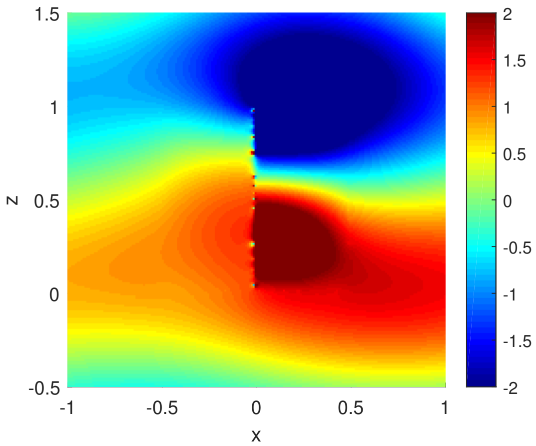

Figure 6. Colormap of the y-component of the magnetic near field H_y (V/m) plotted along the x and z axes at $y = 0.5$

Fig. 6 and Fig. 7 allow to inspect that the continuity of these field componentes agree with the jump conditions. The effect of introducing a perfectly conducting sheet is clearly demonstrated in Fig. 8, in which the e_z and h_y components are plotted as one traverses the system along the x-axis at $(y = 0.5, z = 0.5)$.

References

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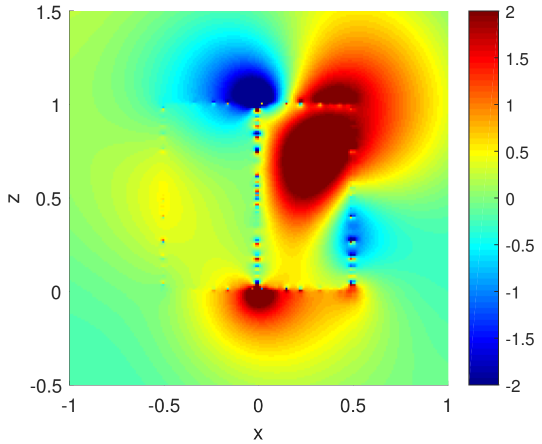

Figure 7. Colormap of the z component of the electric near field E_z (V/m) plotted along the x and z axes at $y = 0.5$

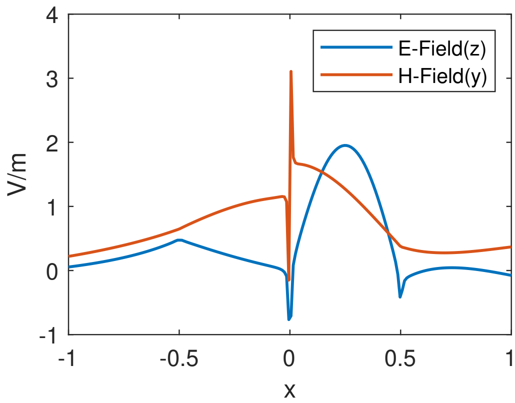

Figure 8. Along the x-axis, the z-component of the electric field can be seen to be continuous and tend to zero as the sheet is approached. The y-component of the magnetic field is continuous, except at the sheet, where it jumps proportionate to the induced current.

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