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On the Solution of the Brain-to-Skull Contrast Breakdown with a New Integral Scheme: Mathematical Framework and Medical Application Scenarios

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Electroencephalography (EEG) from scalp potentials is one of the primary strategies for imaging the brain activity and plays a crucial role in diagnostic and treatment of several, often severe, neuro-pathologies including epilepsy, Parkinson's, Alzheimer's, and lock-in syndromes. Its most advanced incarnation, high-resolution EEG, is heavily leveraging advanced electromagnetic modelling requiring iterated solutions of the EEG forward problem: an electromagnetic mapping from the currents in the (inhomogeneous and aninotropic) brain volume to the electric potentials on the scalp. In the forward problem, both the neural sources and the head should be modeled as accurately as possible with numerical solvers to maximize the precision of the localization. The solvers employed in the EEG forward problem are often based on integral equations since they do not require any discretization outside the object. Depending of the accuracy of the conductivity profile at disposal, the modeling should be performed with surface or volume formulations. The head is composed of some conductive tissues (e.g. white matter, cerebrospinal fluid) and a weakly conducting layer, the skull tissues.

Lamentably, standard forward problem solvers suffer from ill-conditioning and numerical instabilities due to the low conductivity of the skull with respect to the other tissues, which causes slow convergence and dramatic losses of accuracy in the solution. The most spread scheme to solve this problem is the isolated skull approach [1]. It consists in dividing the original problem into two sub problems that do not suffer from instabilities. These strategies, however are computationally cumbersome and limited in scope to surface integral formulations which operate in objects with piecewise constant conductivity, thus preventing the inclusions of anisotropies and inhomogeneities in the model, both of which are fundamental for high fidelity neuroimaging. This notwithstanding, when an accurate conductivity profile of the head is available, surface integral equations formulations are not enough to handle inhomogeneities and anisotropies and one has to resort to volume methods. However, as their surface counterparts, volume formulations suffer from the high-contrast breakdown.

In this work we address and solve the above mentioned problems by proposing a new volume integral equation which is the first integral formulation capable of handling anisotropies and inhomogeneities of the brain media while being immune from the numerical problems originating from the high conductivity contrast between the skull and the other tissues in the head. The new scheme is obtained by introducing a novel set of volume quasi-Helmholtz projectors. Their scaling by the material permittivity matrix allows for the re-balancing of the equation when applied to inhomogeneous scatterers and thereby makes the proposed method accurate and stable for high-contrast objects.

This contribution not only introduces the new equation, but also equips it with a rigorous theoretical framework with all key theorems justifying the new equation's performance. The theoretical framework will also be corroborated by numerical results in both canonical and real case medical scenarios illustrating the impact of the new formulation for the medical practice.

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References

[1] J. Sarvas, "Basic mathematical and electromagnetic concepts of the biomagnetic inverse problem," *Physics in Medicine and Biology*, vol. 32, no. 1, pp. 11–22, jan 1987.