URSI GASS 2021, Rome, Italy, 28 August - 4 September 2021

Improving the NeQuick topside model through COSMIC/FORMOSAT-3 Radio Occultation data

Alessio Pignalberi*(1), Michael Pezzopane(1), David R. Themens (2), Haris Haralambous(3), Bruno Nava(4), and Pierdavide Coïsson(5)
(1) Istituto Nazionale di Geofisica e Vulcanologia, Rome, Italy
(2) Department of Physics, University of New Brunswick, Fredericton, New Brunswick, Canada
(3) Frederick University, Nicosia, Cyprus
(4) The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy
(5) Université de Paris, Institut de physique du globe de Paris, CNRS, Paris, France

Abstract

In this paper, the NeQuick topside formulation is mathematically inverted to derive a fully analytical expression of the topside scale height as a function of the electron density and F2-layer peak parameters. By fitting the NeQuick topside scale height formula to calculated topside scale height values, it is possible to obtain new calibrated values of H_0, r, and g topside parameters for a better description of the topside electron density profile.

This new methodology has been applied to a selected and very reliable dataset of COSMIC/FORMOSAT-3 Radio Occultation topside profiles. Statistical analyses strongly support the applied approach in view of a possible application to the entire COSMIC dataset.

1 Introduction

The topside part of the ionosphere extends from the F2-layer peak, corresponding to the ionospheric electron density maximum (*Nm*F2), to the plasmasphere [1]. The topside is characterized by a decrease of the electron density as the ion population smoothly transitions from a region dominated by heavy O^+ ions in the F-layer, to an upper region dominated by less heavy H^+ and He^+ ions. This behavior is usually described by means of monotonically decreasing analytical functions dependent on a parameter called topside scale height [2].

A reliable model of the topside ionosphere is one of the most difficult tasks when modeling the vertical electron density profile of the ionosphere, because the instruments commonly used to sound the ionosphere often are only capable of sounding the region below the height (*hm*F2) of the F2-layer electron density peak. This is why the most established ionospheric models, the International Reference Ionosphere (IRI) [3] and the NeQuick models [4], are not always able to properly represent the real features of the topside ionosphere [5]; therefore, further studies aimed at improving their topside representation are needed [6]. The NeQuick topside formulation (which is also the recommended one of IRI's three topside options) describes the topside electron density profile by means of a semi-Epstein layer with a height dependent empirically determined topside scale height. The NeQuick topside scale height depends indeed on three parameters: H_0, r, and g. H_0 is the scale height at the peak; r is a parameter which allows restricting the scale height increase at higher altitudes and g is the height gradient for the scale height H_0. [7] demonstrated that the NeQuick topside model does not succeed in capturing the curvature of the topside at high and mid latitudes. They show that the parameters used by NeQuick and IRI to model H_0 are not able to fully describe the seasonal and diurnal variability of H_0 in these regions. Furthermore, they confirmed that a revision of the choice of r and g parameters is necessary for application to mid- and high-latitude regions. For their purposes, an r value of 20 and a g value of 0.2024 were found to optimally represent the curvature of the topside profile.

Also [8] proposed an improvement to the topside representation of the NeQuick model through the implementation of a new analytical formulation of the H_0 parameter. To accomplish this task, they fitted the NeQuick topside analytical function through two anchor points: the F2-layer absolute electron density maximum and the electron density value as measured by Swarm satellites.

In this work, we show how to analytically invert the semi-Epstein formulation (on which the NeQuick topside is based) to derive the topside scale height as a function of measured parameters. Then, a reliable Constellation Observing System for Meteorology, Ionosphere, and Climate (COSMIC) Radio Occultation (RO) profiles dataset is considered to investigate the topside scale height behavior. Finally, it will be shown how to fit the NeQuick topside scale height formula to COSMIC derived ones, and how it is possible to obtain calibrated values of the NeQuick topside parameters H_0, r and g.

2 The NeQuick topside model

The NeQuick topside analytical formulation [4] consists of a semi-Epstein layer describing the topside electron density N_e as a function of the height h, starting from the *Nm*F2 value at the *hm*F2 height:

$$N_e(h) = 4Nm\mathrm{F2}\frac{\exp\left(\frac{h-hm\mathrm{F2}}{H}\right)}{\left[1+\exp\left(\frac{h-hm\mathrm{F2}}{H}\right)\right]^2}; \qquad (1)$$

the electron density decrease with height is driven by a modeled scale height H,

$$H(h)=H_0\left[1+\frac{rg\left(h-hm\mathrm{F2}\right)}{rH_0+g\left(h-hm\mathrm{F2}\right)}\right]. \qquad (2)$$

(2) shows that NeQuick describes the scale height as a function of three empirically deduced parameters: H_0, g, and r. H_0 is the value assumed by the scale height at the F2-layer peak height ($H=H_0$ when $h=hm$F2), whereas $g=0.125$ and $r=100$ [4].

While r and g parameters have been empirically set to constant values, H_0 is modeled as a function of the bottomside thickness parameter, solar activity index, and other bottomside parameters [4].

3 COSMIC/FORMOSAT-3 Radio Occultation data

COSMIC/FORMOSAT-3 is a constellation made up of six microsatellites launched on 15 April 2006 into a circular orbit (with 72° of inclination) at about 800 km of altitude, and a separation angle of 30° in longitude between neighboring satellites. The mission is a collaborative project between the National Space Organization in Taiwan and the University Corporation for Atmospheric Research (UCAR) in the United States. Each satellite carries a Global Positioning System (GPS) RO receiver capable of measuring the phase delay of radio waves from GPS satellites as they are occulted by the Earth's atmosphere and thus providing an accurate determination of the ionospheric vertical electron density profile.

The calculation of the topside scale height requires very reliable topside profiles. In order to perform our investigation using topside RO profiles under the best possible collocation conditions with corresponding ionosondes, in space and time, we compiled a very reliable dataset by selecting RO COSMIC profiles collocated (within 1° in both latitude and longitude) and simultaneous (within 7.5 minutes) with ionosonde measured profiles for which both $Nm\mathrm{F2}_{\mathrm{COSMIC}} \approx Nm\mathrm{F2}_{\mathrm{Ionosonde}}$ and $hm\mathrm{F2}_{\mathrm{COSMIC}} \approx hm\mathrm{F2}_{\mathrm{Ionosonde}}$ matched within 5%. Ionosonde data used in this study were downloaded from the Digital Ionogram DataBASE by means of the SAO Explorer software developed by the University of Massachusetts, Lowell. The electron density profiles were the result of a manual scaling of the ionograms to maximize the accuracy of the data used.

In standard RO data inversion, the error is systematically accumulated from the top of the RO inverted profile to the bottom; thus, when $Nm\mathrm{F2}_{\mathrm{COSMIC}}$ matches $Nm\mathrm{F2}_{\mathrm{Ionosonde}}$ it is expected that the full topside of the RO profile is correctly estimated [9].

According to these constraints, we were able to select 382 profiles over selected ionosonde stations, from 2006 to 2015. The spatial distribution and the number of selected COSMIC profiles are provided in the bubbleplot of Figure 1.

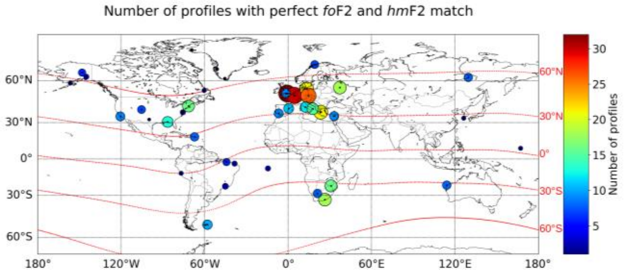

Figure 1. Bubbleplot representing the spatial distribution and the number of selected COSMIC RO profiles. Red dotted lines represent Quasi-Dipole magnetic latitude isolines.

4 Topside modeling through COSMIC RO data

By following [10], to calculate topside scale height values, the Epstein function (1) is analytically inverted as follows:

$$(1)\Rightarrow \frac{N_{\mathrm{e}}(h)}{4Nm\mathrm{F2}}=\frac{\exp\left(\dfrac{h-hm\mathrm{F2}}{H}\right)}{\left[1+\exp\left(\dfrac{h-hm\mathrm{F2}}{H}\right)\right]^2}\,; \qquad (3)$$

by making the following variables change

$$\begin{cases} t=\exp\left(\dfrac{h-hm\mathrm{F2}}{H}\right) \\ \alpha=\dfrac{N_{\mathrm{e}}(h)}{4Nm\mathrm{F2}} \end{cases}, \qquad (4)$$

(3) becomes

$$\alpha=\frac{t}{1+2t+t^2}\Rightarrow \alpha t^2+\left(2\alpha-1\right)t+\alpha=0\,; \qquad (5)$$

(5) is a quadratic equation in the variable t that can be easily solved to obtain the following two solutions:

$$\begin{cases} t_1(h)=\dfrac{1}{N_{\mathrm{e}}(h)}\left[\left(2Nm\mathrm{F2}-N_{\mathrm{e}}(h)\right)+2\sqrt{Nm\mathrm{F2}^2-N_{\mathrm{e}}(h)\cdot Nm\mathrm{F2}}\right] \\ t_2(h)=\dfrac{1}{N_{\mathrm{e}}(h)}\left[\left(2Nm\mathrm{F2}-N_{\mathrm{e}}(h)\right)-2\sqrt{Nm\mathrm{F2}^2-N_{\mathrm{e}}(h)\cdot Nm\mathrm{F2}}\right] \end{cases}. \qquad (6)$$

By inverting t from (4) to obtain the scale height H we get

$$H_{1,2}(h)=\frac{h-hm\mathrm{F2}}{\ln\left[t_{1,2}(h)\right]}\,; \qquad (7)$$

analytically, two solutions for H are possible: the one corresponding to t_1 and the other one corresponding to t_2. It can be easily verified from (6) that $t_1 \geq 1$ and $0 \leq t_2 \leq 1$ in the topside (where $N_{\mathrm{e}}(h)\leq Nm\mathrm{F2}$); by putting t_1 and t_2 in (7), we can verify that in the topside (where $h \geq hm\mathrm{F2}$) $H_1 \geq 0$ and $H_2 \leq 0$. Even though both are solutions mathematically acceptable, we consider t_2 unacceptable because it produces negative values of H.

Thus, the topside scale height which will be used in this work is the one corresponding to t_1, which for simplicity we name H_{Epstein} from now on, as follows:

$$H_{\mathrm{Epstein}}(h)=\frac{h-hm\mathrm{F2}}{\ln\left\{\dfrac{1}{N_{\mathrm{e}}(h)}\left[\left(2Nm\mathrm{F2}-N_{\mathrm{e}}(h)\right)+2\sqrt{Nm\mathrm{F2}^2-N_{\mathrm{e}}(h)\cdot Nm\mathrm{F2}}\right]\right\}}$$

.(8)

By applying (8), it is possible to calculate the topside scale height H_{Epstein} for each height h by using measured $N_e(h)$, NmF2, and hmF2 values [10].

After that, the NeQuick topside scale height formula (2) is fitted to calculated H_{Epstein} values to obtain new calibrated H_0, r, and g values. An example of the applied methodology is shown in Figure 2.

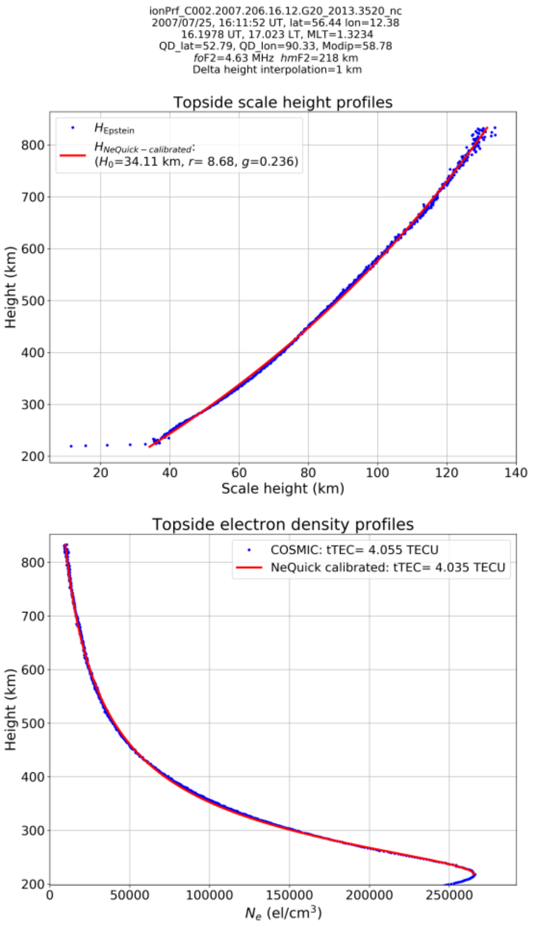

Figure 2. (top panel) Topside scale height values (H_{Epstein}, blue points) obtained from the COSMIC measured profile shown in the bottom panel, and corresponding modeling by fitting the NeQuick scale height ($H_{\mathrm{NeQuick\ calibrated}}$, red curve). (bottom panel) Topside electron density values (blue points) measured by COSMIC and (red curve) modeled by using the topside scale height $H_{\mathrm{NeQuick\ calibrated}}$. The COSMIC profile is the one measured on 2007/07/25 at 16:11:52 UT (Universal Time) at Lat=56.44° N and Lon=12.38° E.

5 NeQuick calibrated approach applied to selected COSMIC profiles

For each of the selected COSMIC RO profiles, the topside total electron content (tTEC) is calculated by integrating electron density values from hmF2 to the satellite height, as follows:

$$\mathrm{tTEC}_{\mathrm{measured}} = \int_{hm\mathrm{F2}}^{h_{\mathrm{COSMIC}}} N_{\mathrm{e,COSMIC}}\, \mathrm{d}h \,. \qquad (9)$$

Moreover, each topside RO profile provides measured values of NmF2 and hmF2 that are used to model the topside profile through (1) with new calibrated H_0, r, and g values obtained by fitting (2) to H_{Epstein} values calculated through (8) for the considered RO profile.

tTEC modeled values are then calculated as follows:

$$\mathrm{tTEC}_{\mathrm{mod\,eled}} = \int_{hm\mathrm{F2}}^{h_{\mathrm{COSMIC}}} N_{\mathrm{e,NeQuick\ calibrated}}\, \mathrm{d}h \,. \qquad (10)$$

Then, tTEC Root Mean Square Error (RMSE) and Normalized Root Mean Square Error (NRMSE) values are calculated, expressed in TECU (1 TECU=10^{16}el/m^2) and in percentage, for the entire selected COSMIC dataset:

$$\mathrm{RMSE\ [TECU]} = \sqrt{\frac{\sum_{i=1}^{N}\left(\mathrm{tTEC}_{\mathrm{mod\,eled},i} - \mathrm{tTEC}_{\mathrm{measured},i}\right)^2}{N}} \,, \qquad (11)$$

$$\mathrm{NRMSE\ [\%]} = \sqrt{\frac{\sum_{i=1}^{N}\left(\frac{\mathrm{tTEC}_{\mathrm{mod\,eled},i} - \mathrm{tTEC}_{\mathrm{measured},i}}{\mathrm{tTEC}_{\mathrm{measured},i}}\cdot 100\right)^2}{N}} \,, (12)$$

where N=382.

We have obtained the following statistical values: RMSE=0.064 TECU and NRMSE=0.716 % using all 382 COSMIC profiles.

Figure 3 show the histogram of residuals between topside tTEC values modeled through the NeQuick calibrated formulation and COSMIC measured ones, and a scatter plot of the modeled and measured tTEC values. Figure 3 shows that the distribution of residuals is well peaked around the zero (residuals mean = 0.001 TECU) with a very low dispersion (residuals standard deviation = 0.064 TECU). Similar considerations can be drawn from the scatter plot which exhibits a one-to-one dependence between measured and modeled tTEC values (slope = 1.001, intercept = -0.008 TECU, Pearson correlation coefficient = 1.0).

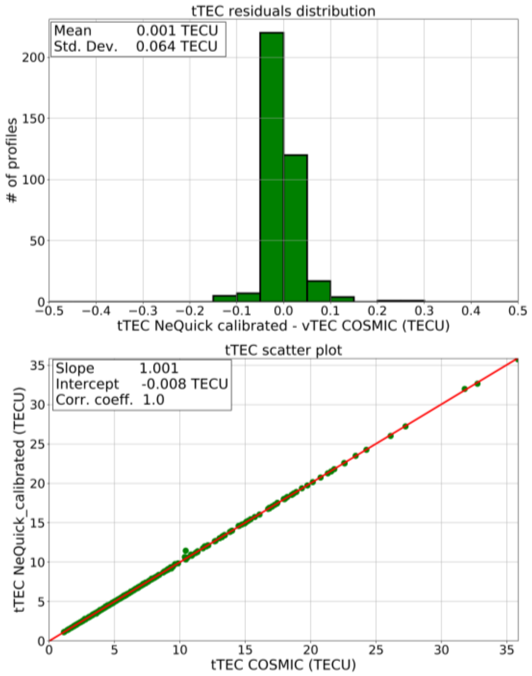

Figure 3. (top panel) Histogram of residuals between tTEC values modeled by the NeQuick calibrated formulation and COSMIC measured ones. (bottom panel) Scatter plot of tTEC values modeled by the NeQuick calibrated formulation and COSMIC measured ones. In red the best linear fit.

Statistics on tTEC values gives an overall picture of performance for the whole topside profile; however, it is also interesting to show how residuals of the electron density are distributed as a function of height. Figure 4 presents a density plot of percentage residuals between modeled ($N_{\mathrm{e,NeQuick\ calibrated}}$) and measured ($N_{\mathrm{e,COSMIC}}$) electron density values as a function of the reduced height z, calculated as follows

$$\text{Electron density relative error } (z)\ [\%] = \frac{N_{\mathrm{e,NeQuick\ calibrated}}(z) - N_{\mathrm{e,COSMIC}}(z)}{N_{\mathrm{e,COSMIC}}(z)} \cdot 100\ ; \qquad (13)$$

this figure shows that most of the percentage error lies within 5% for the whole topside profile probed by COSMIC satellites.

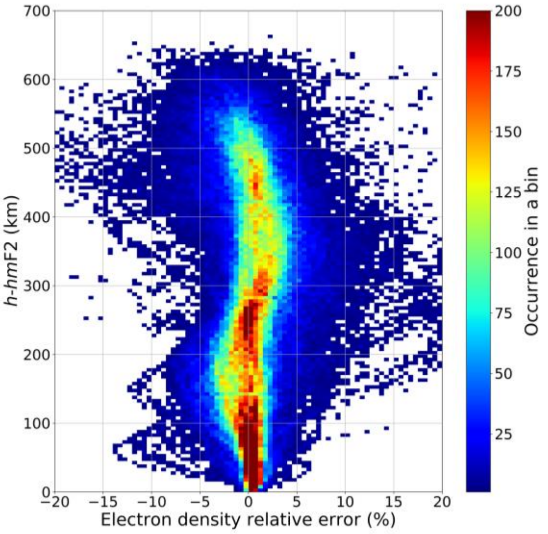

Figure 4. Density plot of residuals percentage between modeled (by the *NeQuick calibrated* procedure) and measured (by COSMIC) electron density values as a function of the reduced height.

6 Conclusions

This paper shows how to calculate topside scale height values by analytically inverting the semi-Epstein layer formulation adopted by the NeQuick model. Applying this new methodology, it is possible to obtain new calibrated values of H_0, r, and g.

The new approach (called *NeQuick calibrated*) has been applied to a selected dataset made by 382 COSMIC RO profiles. Results show that the proposed approach can be potentially applied to get very reliable H_0, g, and r values from both the whole COSMIC (or other missions') profiles dataset and also from Incoherent Scatter Radar profiles.

The application of the *NeQuick calibrated* approach to the COSMIC RO dataset is already being performed in order to obtain a reliable dataset of H_0, r, and g values, so useful for modeling purposes.

Acknowlegements

The authors thank the Telecommunications/ICT for Development (T/ICT4D) Laboratory team of the Abdus Salam International Centre for Theoretical Physics (Trieste, Italy), for developing, maintaining, and making available the NeQuick model (https://t-ict4d.ictp.it/nequick2/nequick-2-web-model).

Thanks are due to the COSMIC/FORMOSAT-3 team for making freely available Radio Occultation data by means of COSMIC Data Analysis and Archive Center (CDAAC, http://cdaac-www.cosmic.ucar.edu/cdaac/products.html).

This publication uses data from ionospheric observatories made available via the public access portal of the Digital Ionogram Database (http://ulcar.uml.edu/DIDBase/) of the Global Ionosphere Radio Observatory in Lowell, MA. The authors are indebted to the observatory directors and ionosonde operators for the significant investments of their time, effort, expertise, and funds needed to acquire and provide measurement data to academic research.

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