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Variable-Resolution SAR Mode With the Principle of Maximum Mutual Information

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Abstract

Based on the related research of a generalized synthetic aperture radar (SAR) modality named variable-resolution (VR) SAR, we formulate the optimization problem of VR SAR based on the principle of maximum mutual information. First, the information content of a specific scene is defined by modeling its distributed scattering as a stochastic process, and the mutual information between scenes and the observed SAR image can be derived. Then, we construct an optimization problem to maximize the mutual information by solving for the optimal beam manipulation scheme of the VR SAR mode. The potential advantage of VR SAR mode is to maximize the efficiency of data acquisition while extracts as much information from scenes as possible. A mathematical model of VR SAR mode with the principle of maximum mutual information is established, and the feasibility and merits of the method are demonstrated through a 1-D simulation.

1 Introduction

Since its inception, synthetic aperture radar (SAR) has proven to be a reliable technique to obtain high-resolution images for earth observation under all weather conditions, day and night. Conventional SAR systems must balance the tradeoff between azimuth resolution and scene size. Recently coding metasurface antennas have emerged as a reliable platform for existing SAR systems and alternative SAR modalities [1, 2].

With the development of programmable metasurface antennas, a generalized SAR mode called variable-resolution (VR) SAR has been proposed in [3]. It can achieve different azimuth resolutions for different ROIs in the azimuth dimension. VR SAR mode is a generalization of the traditional stripmap and spotlight modes. Inspired by the basic principle of spotlight SAR mode, different azimuth resolutions correspond to different synthetic aperture lengths and integration angles. The stripmap mode is a special VR SAR case in which the beam patterns are uniform across the entire trajectory, and the spotlight mode steers the narrow beam constantly towards the ROI along the trajectory. Compared with stripmap SAR, the VR SAR mode makes more efficient and flexible use of beam pattern illumination resources to achieve higher resolution in more interesting regions while reducing the resolution of low-priority regions. In contrast to spotlight SAR, VR SAR is capable of imaging multiple ROIs simultaneously with multiple beams. In addition, the utilization of dynamic patterns would allow us to choose inhomogeneous and variable pulse repetition frequencies (PRFs), and the optimization of PRF has been conducted to obtain a relative larger swath width and a small sample data volume in [3].

The goal of the radar imaging process is to extract information about the target scene from the received data. SAR images are formed by radar signals that are backscattered from ground targets. It is expected that SAR images will maximally reflect the accurate information of the target scenes in order to be used for the postprocessing steps, such as target detection and recognition. In information theory, the interdependence between different random variables is measured by mutual information [4]. Therefore, we choose the mutual information as the objective function and formulate the VR SAR imaging mode with the principle of maximum mutual information in this paper. We conduct theoretical simulation to evaluate the feasibility of the VR SAR mode with the principle of maximum mutual information.

2 Methodology and Derivation

For a general ground target scene, its information can be quantified by the stochastic process modeling of its scattering field. We define the information contained in the ROI according to the stochastic spatial distributions of the coefficients. In addition, we evaluate the VR SAR image quality by the mutual information between the ROI scattering coefficients and the obtained SAR image. Under limited beam resources, the optimal VR imaging scheme can be obtained by maximizing the mutual information between the target scene and SAR image. Then, the optimal imaging scheme can guide the manipulation of diverse radiation patterns and thus allocate different azimuth resolutions to different ROIs.

In this paper, we employ the principle of maximum mutual information to allocate the dynamic beam resources so that the optimal adaptive resolution can be achieved. The basic idea is to optimize the dynamic beam patterns along the trajectory to maximize the mutual information between the scene and the observed SAR image. Thus, we first need to establish a rational definition of the mutual information between an unknown scene and the observed SAR image and then formulate an optimization problem to obtain the maximum mutual information. The observed SAR image is analytically related to the target scene, which can be represented by a stochastic distribution of point scatterers in the imaging plane. As an imaging tool, SAR is mainly concerned with obtaining scatterer information, i.e., the scattering amplitude $\boldsymbol{a}(x,y)$ of the scatterers located at different positions. First, we make the following hypotheses to simplify the model:

1) The 2-D ROI is discretized into M regions. Each region is infinitely meshed into N_r grid cells in the range direction and N_a grid cells in the azimuth direction; i.e., each region is composed of $N_r \times N_a$ grid cells: $N_r \to \infty$, $N_a \to \infty$.

2) The spatial structures and statistical distribution of the scene are regarded as prior knowledge.

3) The scattering amplitude is a 2-D stationary Gaussian process, which is independent of each other in two direction.

The distributed scattering amplitude $\boldsymbol{a}(x,y)$ is modeled as a 2-D discrete generalized stationary stochastic process

$$\boldsymbol{a}(x,y) \overset{def}{=} \boldsymbol{a}(x_0+x\Delta x, y_0+y\Delta y) \tag{1}$$

where Δx and Δy are the step size of one azimuth grid cell and range grid cell, x and y denote the index of a azimuth grid cell and range grid cell, and x_0 and y_0 are the position of the first grid cell in the azimuth direction and range direction, respectively. Note that a grid cell can be simply defined as the minimally concerned unit in VR imaging. The scattering amplitude at one particular position is a random variable obeying a certain distribution. It is known that the discrete Fourier transform (DFT) of the 2-D discrete random process $\boldsymbol{a}(x,y)$ is also a 2-D discrete random process [5]. Then, the scattering coefficient in the wavenumber domain is

$$\boldsymbol{A}(u,v) = \sum_{x=0}^{N_a-1}\sum_{y=0}^{N_r-1} \boldsymbol{a}(x_0+x\Delta x, y_0+y\Delta y)\mathrm{e}^{-j\frac{2\pi}{N_a}ux}\mathrm{e}^{-j\frac{2\pi}{N_r}vy}. \tag{2}$$

The DFT of a generalized stationary process is nonstationary white noise with the autocorrelation function [5]

$$\begin{aligned} R_{\boldsymbol{A}}(\Delta u, \Delta v) &= E\left[\boldsymbol{A}(u,v)\boldsymbol{A}^*(u+\Delta u, v+\Delta v)\right] \\ &= 4\pi^2 S_{\boldsymbol{a}}(u,v)\delta(\Delta u, \Delta v) \end{aligned} \tag{3}$$

where $S_{\boldsymbol{a}}(u,v)$ is the power spectrum of $\boldsymbol{a}(x,y)$, and $\delta(u,v)$ is the Dirac delta function. $S_{\boldsymbol{a}}(u,v)$ is expressed as

$$S_{\boldsymbol{a}}(u,v) = S_{\boldsymbol{a}}^c(u,v) + 4\pi^2\mu_{\boldsymbol{a}}^2\delta(u,v) \tag{4}$$

where $S_{\boldsymbol{a}}^c(u,v)$ is the covariance spectrum of $\boldsymbol{a}(x,y)$, $\mu_{\boldsymbol{a}}$ is the mean of $\boldsymbol{a}(x,y)$. Since $\boldsymbol{a}(x,y)$ is a 2-D stationary Gaussian process, its autocorrelation coefficient can be written in the form

$$\rho_{\boldsymbol{a}}(\tau\Delta x, \eta\Delta y) = \exp\left(-\frac{|\tau\Delta x|}{l_x} - \frac{|\eta\Delta y|}{l_y}\right) \tag{5}$$

where l_x and l_y are the correlation length of $\boldsymbol{a}(x,y)$ in the azimuth direction and range direction, respectively. From (5) and the relationship between the covariance function $C_{\boldsymbol{a}}(\tau,\eta)$ and the autocorrelation coefficient, the covariance spectrum is derived as

$$\begin{aligned} S_{\boldsymbol{a}}^c(u,v) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C_{\boldsymbol{a}}(\tau,\eta)\mathrm{e}^{-ju\tau}\mathrm{e}^{-jv\eta}\,\mathrm{d}\tau\mathrm{d}\eta \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \sigma_{\boldsymbol{a}}^2\rho_{\boldsymbol{a}}(\tau\Delta x, \eta\Delta y)\mathrm{e}^{-ju\tau}\mathrm{e}^{-jv\eta}\,\mathrm{d}\tau\mathrm{d}\eta \\ &= \sigma_{\boldsymbol{a}}^2 \frac{2\frac{\Delta x}{l_x}}{(\frac{\Delta x}{l_x})^2+u^2}\frac{2\frac{\Delta y}{l_y}}{(\frac{\Delta x}{l_y})^2+v^2} \end{aligned} \tag{6}$$

where $\sigma_{\boldsymbol{a}}^2$ is the variance of $\boldsymbol{a}(x,y)$.

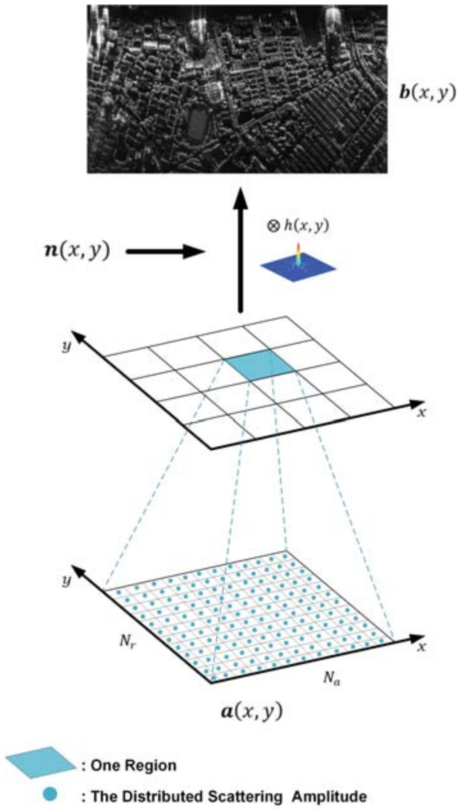

Figure 1. Schematic of the SAR imaging process.

As shown in Figure 1, the SAR imaging process is modeled as a liner system. which is expressed as

$$\boldsymbol{b}(x,y) = \boldsymbol{a}(x,y) \otimes h(x,y) + \boldsymbol{n}(x,y) \tag{7}$$

where "$\otimes$" denotes the convolution operation; $h(x,y)$ is the imaging system transfer function, i.e., the corresponding point spread function (PSF) of the specific region; and $\boldsymbol{n}(x,y)$ denotes the system additive noise. Then, the imaging process in the wavenumber domain can be obtained by taking the DFT of both sides

$$\boldsymbol{B}(u,v) = \boldsymbol{A}(u,v)H(u,v) + \boldsymbol{N}(u,v). \tag{8}$$

Through a series of derivation, the spatial domain mutual information between the target scenes and the obtained SAR images is proven to be equal to the wavenumber domain mutual information $I[\boldsymbol{A}(u,v);\boldsymbol{B}(u,v)]$, which is given by

$$I[\boldsymbol{A}(u,v);\boldsymbol{B}(u,v)] = \frac{1}{2}\sum_{u=0}^{N_a-1}\sum_{v=0}^{N_r-1}\ln\left[1+\frac{S_{\boldsymbol{a}}^c(u,v)\,|H(u,v)|^2}{S_{\boldsymbol{n}}^c(u,v)}\right]. \tag{9}$$

To maximize the mutual information between the M regions and the obtained SAR images, we can formulate the optimization problem as

$$\arg\max \frac{1}{2}\sum_{m=1}^{M}\sum_{u=0}^{N_a-1}\sum_{v=0}^{N_r-1}\ln\left[1+\frac{S_{\boldsymbol{a}_m}^c(u,v)\,|H_m(u,v)|^2}{S_{\boldsymbol{n}}^c(u,v)}\right] \tag{10}$$

3 Simulation and Analysis

To simplify this optimization problem, We conduct a simulation study in which a 1-D(along the azimuth dimension) stochastic scene is to be imaged via VR SAR mode. Then, the optimization problem is described as

$$\begin{aligned} &\arg\max\ \mathbf{GL} \\ &\text{s.t.}\begin{cases} \sum_{m=1}^{M}\ \mathbf{P}(m,t) \le \lfloor \frac{R_0\lambda}{2VS_0}\text{PRF}\rfloor \\ 0 \le \mathbf{P}(m,t) \le 1 \end{cases} \end{aligned} \tag{11}$$

where $\mathbf{G}$ is the slope vector of the mutual information with increasing power spectrum width; $\mathbf{P}(m,t)$ is the illumination matrix; R_0 is the reference slant range; λ is the wavelength; V is the velocity of the antenna platform; S_0 is the span range of each region. The first constraint means that the beamwidth is limited by the predetermined PRF. The ROI is simulated as a Gaussian stochastic process. The whole ROI is subdivided equally into five regions in the azimuth dimension, and each region spans 100 m intervals. Furthermore, each region is composed of 250 grid cells. The random scattering coefficient of each region is modeled as a Gaussian stochastic process with different variances and correlation lengths. We simulate the optimal VR SAR mode and, as a comparison, we also simulate a standard strip map mode. The azimuth sampling positions are forced to be the same for the two modes. Meanwhile, the maximum allowed beamwidth of VR SAR is set to the beamwidth of strip map SAR, which can illuminate up to two regions.

Figure 2(a)-(e) depict the imaging results of the five regions both in strip map SAR mode and VR SAR mode. Figure 2(a) shows the first region, which has the maximum information entropy of the scattering coefficient. The imaging results in VR SAR mode are more consistent with the original scattering coefficient trend, and the details of the scattering coefficient are easily visible, especially the sharp peaks. In comparison, adjacent scatterers appear to be one merged peak, leading to the loss of details and target information. Therefore, in VR SAR mode, the azimuth resolution assigned to this region after optimization is much more suitable and reasonable than the resolution of strip map SAR mode.

Table 1. Mutual Information Comparison

Noise PSD	VR SAR Theoretical (nat)	VR SAR Simulated (nat)	Stripmap SAR Theoretical (nat)	Stripmap SAR Simulated (nat)
0.02	1837.21	1105.68	1390.46	964.71
0.08	1168.53	936.43	912.37	828.17
0.14	950.83	867.57	776.24	755.65
0.20	826.46	821.42	745.43	665.59

As shown in Figure 2(b), the scattering coefficient of the second region fluctuates less than that of the first region. In the second region, the azimuth resolution of the two imaging modalities are almost the same, and the imaging results are both very consistent with the original scattering coefficient. In addition, some tiny speaks can be seen in VR SAR mode but cannot be seen in strip map SAR mode. In Figure 2(d) and (e), although the resolution of VR SAR is worse than that of strip map SAR, the imaging results of both are satisfactory, which indicates that VR SAR mode utilizes fewer beam resources to achieve the same imaging effect. In summary, optimization based on the principle of maximum mutual information is valid. The limited beam resources are allocated reasonably to each region to ensure that all the regions are well imaged, extracting maximal scene information and details.

We generate 3000 simulated scene samples from the same stochastic process and image each sample scene four times in both VR SAR mode and strip map SAR mode under different noise power spectral densities (PSDs). We calculate the simulated mutual information according to

$$I[\boldsymbol{a}(x);\boldsymbol{b}(x)] = \frac{1}{2}\ln\left[\frac{|\Sigma_a|\,|\Sigma_{\boldsymbol{b}}|}{\left|\begin{bmatrix}\Sigma_{\boldsymbol{a}} & \Sigma_{\boldsymbol{ab}}\\ \Sigma_{\boldsymbol{ba}} & \Sigma_{\boldsymbol{b}}\end{bmatrix}\right|}\right] \tag{12}$$

where Σ is the covariance matrix. Table 1 shows that as the noise PSD continues to increase, th mutual information acquired in the two modes constantly decreases. Additionally, when the noise PSD value is the same, the VR SAR mode can extract more mutual information than the strip map SAR mode, regardless of the theoretical value or simulated value.

4 Conclusion

This paper formulate the optimization problem of VR SAR with the principle of maximum mutual information. It aims to obtain the maximum mutual information between target scene and SAR image by optimally allocating the dynamic beam patterns. The feasibility and imaging effect are

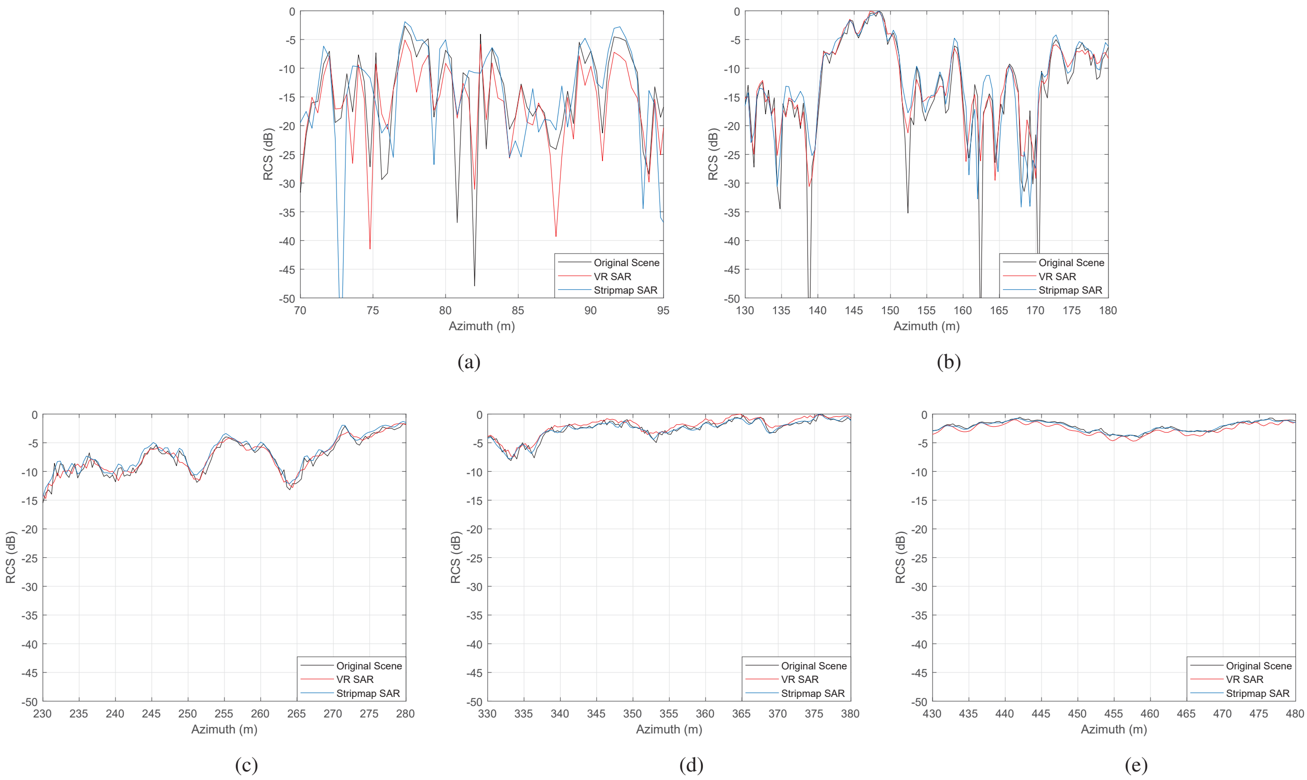

Figure 2. Simulated images of a 1-D scene conducted with strip map SAR mode and VR SAR mode. (a)-(e) Partial imaging results of five regions in sequence.

demonstrated via theoretical simulation. The 2-D equivalent experiment of VR SAR mode with the principle of maximum mutual information will be discussed in our future work.

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