In the past few years, novel approaches to radar signal processing have been introduced which allow the radar signal detection and parameter estimation using much smaller number of measurements than required by Nyquist sampling. These systems exploit the fact that the target scene is sparse facilitating the use of recent advances in compressed sensing methods. This tutorial will introduce recent developments in reduced-rate sampling that break the link between common radar design trade-offs such as range resolution and transmit bandwidth; dwell time and Doppler resolution; spatial resolution and number of antenna elements; continuous-wave radar sweep time and range resolution. For each of these ideas, we present state-of-the-art hardware prototypes that we have designed and developed to demonstrate the real-time feasibility. We examine extensions to diverse applications such as cognition, spectral coexistence, matrix completion, autonomous driving, ground penetration radar, multiple-input-multiple-output and synthetic aperture radars. Finally, we connect the cognition in radars to some interesting applications of deep learning.