URSI GASS 2021, Rome, Italy, 28 August - 4 September 2021

The History of Radiation from Leaky-Wave Antennas

Walter Fuscaldo(1), Alessandro Galli(2), and David R. Jackson*(3)
(1) Consiglio Nazionale delle Ricerche, Istituto per la Microelettronica e Microsistemi (CNR-IMM), 00133 Rome, Italy
(2) Dept. of Information Engineering, Electronics and Telecommunications, Sapienza University of Rome, 00184 Rome, Italy
(3) Dept. of Electrical and Computer Engineering, University of Houston, Houston, TX 77204-4005, USA

Abstract

Leaky-wave antennas (LWAs) are now widespread and designed over a wide frequency range, from microwaves up to optics. One of the attractive features of LWAs is their simple design process as a consequence of the accurate analytical description of their radiating features. In this work, we review from a historical perspective the milestones that characterized the mathematical description of radiation from LWAs, starting from the pioneering contributions of N. Marcuvitz and A. A. Oliner, until the more recent developments.

1 Introduction

In the 1950s, Nathan Marcuvitz set the foundation of leaky-wave theory, recognizing that, in open problems, *nonspectral* solutions, although not generally constituting a complete set of eigensolutions, may characterize the dominant contribution of a field representation [1]. Such nonspectral solutions are now well-known in electromagnetics as *leaky waves*. The theory of leaky waves originally set by Marcuvitz and then extensively developed by Arthur A. Oliner and his collaborators [2, 3], is now profitably used to rigorously describe radiation from a wide class of antennas commonly known as *leaky-wave antennas* (LWAs) [4–6].

In this work, we review from a historical perspective the progress that has been made to obtain an accurate description of the radiating properties of different types of LWAs. Starting from the first formulas derived for one-dimensional (1-D) LWAs with unidirectional excitation under the hypothesis of infinite aperture size, we discuss how these formulas have been recently improved to account for the effect of the aperture truncation and extended to the case of bidirectional excitation, as well as the special case of endfire radiation. Formulas for 2-D LWA patterns and their relationship with 1-D LWA patterns are also reviewed.

2 History of Leaky Waves

The history of leaky waves is closely connected with their *improper* nature. This aspect initially aroused some skepticism about the physical significance of leaky waves. First Marcuvitz, and then Oliner, solved the debate developing a rigorous mathematical framework [2, 3] that is today recognized as *leaky-wave theory*. A simple ray-optics interpretation of a canonical 1-D leaky-wave structure reveals that a leaky-wave field characterized by a complex wavenumber $k_z = \beta - j\alpha$ (being z the axis of propagation) exhibits an exponentially-growing behavior in the transverse direction only in a limited angular region marked by a shadow boundary that occurs at an angle

$$\theta_0 = \arcsin(\beta/k_0), \tag{1}$$

measured from the vertical axis, k_0 being the free-space wavenumber. In the wedge-shaped region beyond angle θ_0, which includes the aperture, the fields are exponentially increasing vertically but exponentially decaying radially. A physical leaky wave that is dominant on a significant part of an antenna aperture will then determine the far-field radiation pattern via Fourier transforming the aperture field of the leaky wave. Therefore, the structure will radiate a narrow beam at angle θ_0 determined solely by the normalized phase constant β/k_0, with a beamwidth determined solely by the normalized attenuation constant (or *leakage rate*) α/k_0; the smaller the leakage rate, the narrower the beamwidth.

3 1-D Leaky-Wave Antennas

3.1 Classification

One-dimensional LWAs are based on a 1-D guiding structure, and are commonly classified based on their physical structure as *uniform*, *quasi-uniform*, or *periodic*, and also on the excitation type, i.e., *unidirectional* or *bidirectional* (see Fig. 1) [4, 5]. Recent progress has been made for describing radiation from 1-D LWAs, in the case of either unidirectional or bidirectional excitation. The formulas developed apply to all three types of LWAs, with β usually representing the phase constant of the radiating $n = -1$ harmonic in the periodic case.

3.2 General Radiating Features

One-dimensional unidirectional LWAs typically produce a scannable beam that can theoretically point from broadside to endfire (see Fig. 1, left), but in practice radiation

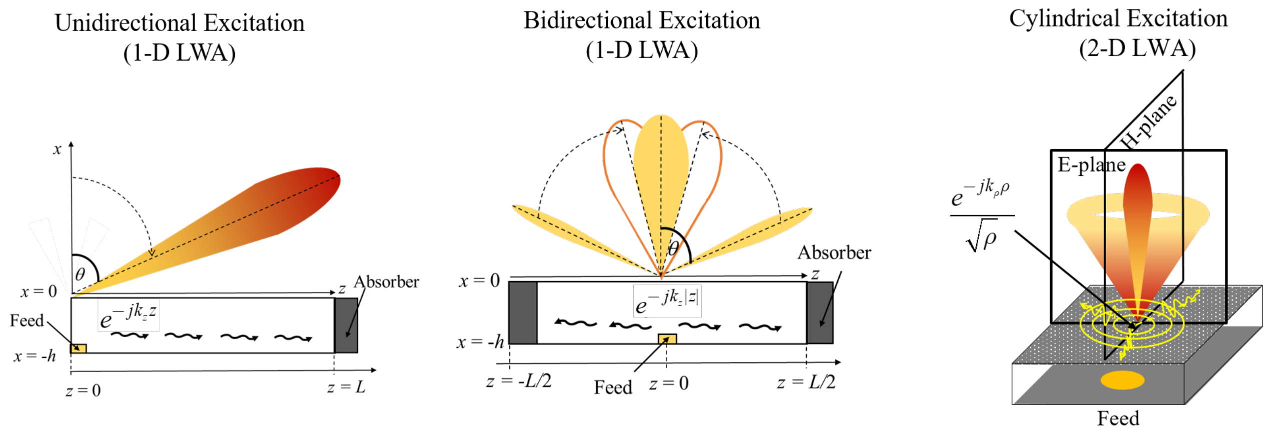

Figure 1. Examples of 1-D LWAs with either unidirectional (left) or bidirectional (middle) excitation, and a 2-D LWA (right).

at broadside and endfire is rather poor [6]. In order to efficiently radiate at broadside, bidirectional excitation is preferred [5] (Fig. 1, middle), whereas 1-D endfire LWAs have been developed (see, e.g., [6] and refs. therein) to efficiently radiate at endfire. 1-D bidirectional LWAs (Fig. 1, middle) normally produce a dual beam pattern with beam angles pointing at $\theta = \pm\theta_0$. As θ_0 approaches 0° (broadside) the two beams approach each other and merge into a single beam, which has a maximum at broadside (see Fig. 1, middle). As a result, a broadside beam can be produced by a 1-D bidirectional LWA, whose radiation properties are actually similar to those of the 2-D LWAs radiating from a cylindrical leaky wave (Fig. 1, right) [5].

4 Early Formulas for 1-D LWAs

T. Tamir and A. A. Oliner rigorously described the physics and the mathematics of leaky waves in their pioneering work in two parts [2,3]. In [3], the patterns of 1-D LWAs for the cases of either unidirectional or bidirectional excitation were derived under the hypothesis of an infinite-aperture z-invariant (nontapered, i.e., $\alpha(z) = \alpha$) 1-D structure. For the unidirectional case, the normalized power pattern for the *infinite-aperture* case is given by

$$P(\theta) = \frac{1}{(\beta - k_0 \sin\theta)^2 + \alpha^2}. \quad (2)$$

The beam angle is given exactly by (1) (the same angle θ_0 that corresponds to the shadow boundary in the near field). A simple analytical formula was then proposed to evaluate the half-power beamwidth $\Delta\theta$ of 1-D unidirectional LWAs, which reads

$$\Delta\theta = 2\hat{\alpha} \sec\theta_0, \quad (3)$$

where the 'hat' denotes normalization with respect to k_0. This formula is accurate only for small beamwidths, i.e., for $\hat{\alpha} \ll 1$. Later on, Oliner proposed a formula for the *finite-aperture* case, which reads

$$\Delta\theta = N_O \sec\theta_0 / (L/\lambda_0), \quad (4)$$

where L/λ_0 is the antenna length L normalized to the vacuum wavelength λ_0, and N_O is a constant that depends on the radiation efficiency and the aperture-type; for a uniform-aperture LWA with a 90% radiation efficiency $N_O = 0.91$ [4]. Note that formula (4) does not smoothly turn into formula (3) as the aperture length L tends to infinity. This is one limitation of the early formulas.

For the infinite-aperture 1-D bidirectional case the normalized power pattern is given by

$$P(\theta) = \frac{\beta^2 + \alpha^2}{[\beta^2 - \alpha^2 - (k_0 \sin\theta)^2]^2 + 4\alpha^2\beta^2}. \quad (5)$$

When the dual beams are pointing off-broadside, Oliner showed that the beamwidth of the two symmetric beams can still be approximately evaluated with (3), whereas the pointing angle is given by [3]

$$\theta_0 = \arcsin\sqrt{\hat{\beta}^2 - \hat{\alpha}^2}, \quad (6)$$

which coincides with (1) for $\hat{\alpha} \ll \hat{\beta}$ (as often occurs in practice). However, for 1-D bidirectional LWAs radiating at *broadside* ($\beta \le \alpha$) a general formula valid in the infinite case was provided only after several decades [7] and reads

$$\Delta\theta = 2\arcsin\left(\sqrt{\hat{\beta}^2 - \hat{\alpha}^2 + \sqrt{2(\hat{\beta}^4 + \hat{\alpha}^4)}}\right). \quad (7)$$

In that work, it was also shown that the point at which the dual beam merges into a single beam occurs when $\beta = \alpha$; this is commonly known as the *beamsplitting condition*, and, in the infinite case also provides the condition for radiating the maximum power density at broadside for Fabry–Perot types of LWAs that use a partially reflecting surface.

5 Recent Formulas for 1-D LWAs

Recent advances have been made in characterizing the beam properties of 1-D LWAs. One advance has been the development of accurate beamwidth formulas for the case of finite-size apertures. The radiating properties of a finite-aperture LWA can be derived from the corresponding pattern function by establishing a beamwidth equation and solving for its roots. In the finite-aperture case, the beamwidth equation is always transcendental, and thus exact analytical solutions do not exist. However, accurate fitting schemes have recently been found to interpolate the numerical solutions so as to provide sufficiently general and accurate analytical expressions in the finite-aperture case. The following subsections will review the main results obtained for 1-D unidirectional and bidirectional LWAs, as well as for the special case of 1-D endfire LWAs.

5.1 Unidirectional Excitation

At first glance, it is clear that (3)–(4) are both singular at $\theta_0 = \pi/2$, and thus inaccurate results are expected as the beam approaches endfire. Moreover, (4) is evidently empirical, and Oliner never provided a general criterion for choosing the correct value of N_O as a function of radiation efficiency, but furnished a few values for relevant cases of interest [4]. A more recent investigation led to the formula provided in [8] and reported here for the readers' convenience:

$$\Delta\theta = 2\left\{\arccos\left\{\hat{\beta} - \frac{2.783}{k_0 L}\left[1 - \tanh\left(0.021\frac{\alpha L}{2}\right)\right]\right.\right.$$
$$\left.\left. -\hat{\alpha}\tanh\left(0.21\frac{\alpha L}{2}\right)\right\} - \arcsin\hat{\beta}\right\}. \quad (8)$$

The radiation efficiency η_r is given by $\eta_r = 1 - \exp(-4a)$, where $a = \alpha L/2$. As opposed to (3)–(4), (8) is no longer singular at endfire. Moreover, (8) no longer requires the beamwidth to be small, and it is accurate for arbitrary antenna lengths and pointing angles, provided that $\hat{\beta} < 1$ and we are not too close to endfire. As we approach endfire, the beam becomes asymmetric, with the half-sided beamwidth on the right side (the side closer to endfire) becoming larger than the half-sided beamwidth on the left side. The beamwidth is thus no longer accurately given by twice the half-sided beamwidth, as assumed in (8). Recently an improved formula was derived that remains accurate down to endfire. This formula is [9]

$$\Delta\theta = \arcsin\left(\hat{\beta} + \frac{t_h(a)}{k_0 L/2}\right) - \arcsin\left(\hat{\beta} - \frac{t_h(a)}{k_0 L/2}\right), \quad (9)$$

where

$$t_h(a) = 1.3915[1 - \tanh(0.021a)] + a\tanh(0.21a). \quad (10)$$

As discussed in [10], in order to achieve maximum directivity at endfire, a 1-D LWA should be designed according to a *modified* Hansen–Woodyard condition, which requires $\hat{\beta} = 1 + \Delta\hat{\beta} > 1$, with $\Delta\hat{\beta}$ depending on both $\hat{\alpha}$ and L. For this purpose, a formula for describing the radiating features of finite-aperture 1-D endfire LWAs has been provided in [11]. The special case of endfire radiation in LWAs has a number of peculiar features. One of the most interesting is that as $\Delta\hat{\beta}$ increases, the beamwidth decreases but the sidelobe level increases. In fact, a point is reached for which the sidelobe has the same intensity as the main peak. The work in [11] not only elucidates these aspects, but also provides accurate formulas for predicting the beamwidth and the sidelobe level of 1-D endfire LWAs for practical values of the radiation efficiency.

5.2 Bidirectional Excitation

Equation (7) only holds for an infinite aperture and for broadside beams, i.e., for $r = \hat{\beta}/\hat{\alpha} \leq 1$. The ratio r between the phase and attenuation constants plays an important role in the beam features of 1-D bidirectional LWAs.

At the beamsplitting condition, i.e., $r = 1$, Eq. (7) predicts a broadside beam with a beamwidth equal to $\Delta\theta = 2\sqrt{2}\hat{\alpha}$. However, it has recently been shown [12] that a kind of broadside beam can actually be obtained from a infinite 1-D bidirectional LWA for $1 \leq r \leq 1 + \sqrt{2}$ as well. In this range, the beam peak is no longer at broadside but at an angle given by (6). However, the radiated power density is not lower than -3 dB relative to the beam peak, and thus the beam can still be considered to be a broadside beam exhibiting *scalloping*. The upper bound $r = 1 + \sqrt{2}$ is called the *dual-beam* condition and represents the scan angle θ_0 for which the power radiated at broadside is at -3 dB relative to the beam peak, and thus we are on the verge of forming a dual-beam pattern. According to the previous definitions, three different radiating regimes can be distinguished: a non-scalloped beam region for $0 < r \leq 1$, a scalloped-beam region for $1 \leq r \leq 1 + \sqrt{2}$, and a dual-beam region for $r > 1 + \sqrt{2}$. Exact formulas for the beamwidth in each radiating regime for 1-D bidirectional LWAs in the infinite case have been provided in [12]. For $r \leq 1$ (non-scalloped broadside beam), the beamwidth is given by Eq. (7). In the region $1 \leq r \leq 1 + \sqrt{2}$ where we have a scalloped broadside beam, the beamwidth is given by

$$\Delta\theta = 2\arcsin(\hat{\alpha}\sqrt{r^2 - 1 + 2r}). \quad (11)$$

The finite-aperture case is even more interesting. Indeed, as originally shown in [13], when a 1-D bidirectional LWA is truncated, the beam peak points at broadside even for $r > 1$; in other words, the nonscalloped region is no longer upper-bounded by $r = 1$, but by a larger value that depends on the radiation efficiency. However, the work in [13] was limited to a numerical analysis, and formulas were not provided for the beamwidth. For this purpose, in [12] analytic formulas were provided for determining how the beamsplitting and the dual-beam conditions evolve as a function of the radiation efficiency; also provided was an accurate beamwidth formula that works for radiation efficiencies as high as 98% (thus covering most practical cases), with $0 < r \leq 5$, for finite-size 1-D bidirectional LWAs radiating at broadside (in either nonscalloped or scalloped regimes). The formula generally reads

$$\Delta\theta = 2\arcsin\left(\frac{t_h(r,a)}{k_0 L/2}\right), \quad (12)$$

where $t_h(r,a)$ is now a function of both r and a (now related to the radiation efficiency through $\eta_r = 1 - \exp(-2a)$). An analytical expression for $t_h(r,a)$ was obtained through numerical fitting and not reported here for brevity (see [12] for further details). In the dual-beam region, the beamwidth formula (9) for a 1-D unidirectional finite-aperture LWA proves to be sufficiently accurate also for the bidirectional finite-aperture case, and thus the beamwidth of finite-aperture 1-D bidirectional LWAs are characterized in any radiating regime.

6 Extension to 2-D LWAs

A 2-D LWA with a cylindrical radially propagating leaky wave produces a conical beam that, as frequency varies, merges into a pencil beam (see Fig. 1, right), provided that both TM and TE leaky modes are suitably excited by a horizontal dipole-like source [14]. (Note that the use of more complex sources has been very recently discussed in [15, 16] in connection with the excitation of higher-order cylindrical leaky waves.) In that case, the TM(TE) leaky mode determines radiation in the E(H)-plane. As can be inferred from Fig. 1, the radiation patterns in the principal planes of a 2-D LWA are quite similar to the pattern of a 1-D bidirectional LWA. In any given azimuth slice of the 2-D LWA pattern, a 1-D bidirectional LWA formula may be used to accurately approximate the pattern of the 2-D LWA.

7 Conclusion

In this contribution we reviewed the radiating features of leaky-wave antennas, with particular emphasis on 1-D LWAs. Starting from the results obtained in the infinite-aperture case by A. A. Oliner more than fifty years ago, we showed how the formulas have been recently improved in accuracy and domain of validity, being also extended to the finite-aperture case. The cases of unidirectional and bidirectional excitation have been treated separately, and a special mention has also been given to the endfire case. Finally, we have shown how a strong connection exists between the physics of 1-D bidirectional leaky waves and cylindrical leaky waves. This allows the 1-D bidirectional formulas that have been recently developed to be extended in an approximate fashion to characterize the beam properties of 2-D LWAs, such as Fabry–Perot LWA structures.

References

[1] N. Marcuvitz, "On field representations in terms of leaky modes or eigenmodes," *IRE Trans. Antennas Propag.*, vol. AP-4, pp. 192–194, Jul. 1956.

[2] T. Tamir and A. A. Oliner, "Guided complex waves–Part I: Fields at an interface," *Proc. Inst. Elec. Eng.*, vol. 110, pp. 310–324, Feb. 1963.

[3] T. Tamir and A. A. Oliner, "Guided complex waves–Part II: Relation to radiation patterns," *Proc. Inst. Elec. Eng.*, vol. 110, pp. 325–334, Feb. 1963.

[4] A. A. Oliner, "Leaky-wave antennas," in *Antenna Engineering Handbook*, R. C. Johnson Ed., New York, NY, USA: McGraw-Hill, 1984, ch. 10.

[5] D. R. Jackson, C. Caloz, and T. Itoh, "Leaky-wave antennas," in *Frontiers in Antennas: Next Generation Design & Engineering*, F. Gross Ed., New York, NY, USA: McGraw-Hill, 2011, ch. 9.

[6] D. R. Jackson, C. Caloz, and T. Itoh, "Leaky-wave antennas," *Proc. IEEE*, Vol. 100, no. 7, pp. 2194–2206, Jul. 2012.

[7] G. Lovat, P. Burghignoli, and D. R. Jackson, "Fundamental properties and optimization of broadside radiation from uniform leaky-wave antennas," *IEEE Trans. Antennas Propag.*, vol. 54, no. 5, pp. 1442–1452, May 2006.

[8] W. Fuscaldo, D. R. Jackson, and A. Galli, "A general and accurate formula for the beamwidth of 1-D leaky-wave antennas," *IEEE Trans. Antennas Propag.*, vol. 65, no. 4, pp. 1670–1679, Apr. 2017.

[9] W. Fuscaldo, D. R. Jackson, and A. Galli, "A general formula for the half-power beamwidth of 1-D unidirectional leaky-wave antennas," *IEEE Int. Antennas Propag. Symp.*, Montreal, Canada, 5–10 Jul. 2020.

[10] E. M. O'Connor, D. R. Jackson, and S. A. Long, "Extension of the Hansen–Woodyard condition for endfire leaky-wave antennas," *IEEE Antennas Wireless Propag. Lett.*, vol. 9, pp. 1201–1204, 2010.

[11] W. Fuscaldo, D. R. Jackson, and A. Galli, "Beamwidth properties of endfire 1-D leaky-wave antennas," *IEEE Trans. Antennas Propag.*, vol. 65, no. 11, pp. 6120–6125, Nov. 2017.

[12] W. Fuscaldo, D. R. Jackson, and A. Galli, "General formulas for the beam properties of 1-D bidirectional leaky-wave antennas," *IEEE Trans. Antennas Propag.*, vol. 67, no. 6, pp. 3597–3608, Jun. 2019.

[13] A. Sutinjo, M. Okoniewski, and R. H. Johnston, "Beam-splitting condition in a broadside symmetric leaky-wave antenna of finite length." *IEEE Antennas Wireless Propag. Lett.*, vol. 7, pp. 609–612, 2008.

[14] A. Ip and D. R. Jackson, "Radiation from cylindrical leaky waves," *IEEE Trans. Antennas Propag.*, vol. 38, no. 4, pp. 482–488, Apr. 1990.

[15] P. Burghignoli, W. Fuscaldo, D. Comite, P. Baccarelli, and A. Galli, "Higher-order cylindrical leaky waves–Part I: Canonical sources and radiation formulas," *IEEE Trans. Antennas Propag.*, vol. 67, no. 11, pp. 6735–6747, Nov. 2019.

[16] P. Burghignoli, D. Comite, W. Fuscaldo, P. Baccarelli, and A. Galli, "Higher-order cylindrical leaky waves–Part II: Circular array design and validations," *IEEE Trans. Antennas Propag.*, vol. 67, no. 11, pp. 6748–6760, Nov. 2019.