Scattering by a Metamaterial Half-Plane on a PEC Support

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Abstract

The scattering mechanism due to an electromagnetic plane wave impacting on a particular planar composite structure is here studied by means of the Uniform Asymptotic Physical Optics approach. The structure is formed by a perfect electric conductor plane hosting a double negative metamaterial half-screen on its upper surface. The Geometrical Optics field and the diffraction contribution due to the surface discontinuity are added to obtain the total field in the surrounding free space. Numerical tests demonstrate the efficiency of the proposed diffracted field to compensate the discontinuities of the Geometrical Optics field.

1 Introduction

A very interesting set of composite matters includes artificial engineered materials having negative real parts of permittivity and permeability at the frequencies of interest. Accordingly, such materials are denoted as double-negative metamaterials (DNG MTMs) and can be assembled by inserting small inclusions in host media or by fixing inhomogeneities to host surfaces. A proper design of these structures permits satisfying electromagnetic requirements, which cannot be obtained by using natural materials. Research activities as well as industrial, spatial and military applications take recently advantage from this opportunity. Therefore, scattering problems concerning DNG MTMs are very attractive from theoretical and application viewpoints. References [1]-[3] can be used to discover definitions, characteristics and applications as well as other references.

As well-known, the Uniform Geometrical Theory of Diffraction (UTD) [4] offers a ray propagation theory incorporating the diffraction contribution and the Geometrical Optics (GO) ones for the scattering evaluation. An alternative approach working in the UTD context has been proposed in recent years to solve diffraction problems also involving DNG MTMs [5]-[10]. Such an analytical procedure is identified as Uniform Asymptotic Physical Optics (UAPO) approach accounting for the PO approximation of equivalent radiating sources. It implements useful approximations and asymptotic techniques to extract the high-frequency diffraction contribution from the scattering integral containing electric and magnetic equivalent surface currents.

The aim of this paper is to apply the UAPO approach to the plane wave diffraction by a composite structure consisting of a perfect electric conductor (PEC) planar support hosting a DNG MTM half-screen on its upper surface (see Fig. 1). The evaluation of the reflection coefficients for both the polarizations is needed to determine the equivalent sources to be used in the radiation integral involving the DNG MTM cover. According to [7] and [8], the equivalent transmission line (ETL) models are used to this target. Note that the DNG MTM half-screens in [5], [6], [9], and [10] have no a PEC support and, therefore, a transmission contribution must be also considered in the GO field.

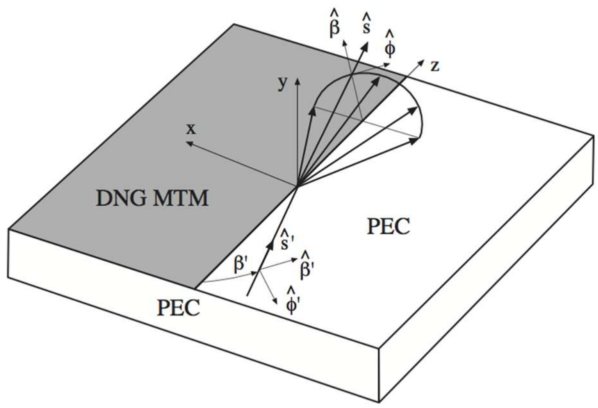

Figure 1. Plane wave diffraction by a DNG MTM half-plane on a PEC support.

2 PO Surface Currents

An infinite PEC support hosting a DNG MTM half-screen on its upper surface is illuminated by an incident plane wave at skew incidence with respect to the linear edge (see Fig. 1). The *xz*-plane and the *z*-axis of a reference co-ordinate system are coincident with the upper surface of the structure and with the DNG MTM edge, respectively. The angles β', ϕ' and the corresponding unit vector $\hat{s}' = -\sin\beta'\cos\phi'\hat{x} - \sin\beta'\sin\phi'\hat{y} + \cos\beta'\hat{z}$ define the incidence direction of incoming plane waves with the electric field expressed in terms of its parallel (||) and perpendicular ($\perp$) components as:

$$\underline{E}^i = \left[E_\perp^i \,\hat{u}_\perp + E_\parallel^i \left(\hat{u}_\perp \times \hat{s}'\right)\right] \exp\left(-jk_0 \hat{s}' \cdot \underline{r}\right) \qquad (1)$$

where k_0 is the free-space propagation constant, $\hat{u}_\perp = (\hat{s}'\times\hat{y})/|\hat{s}'\times\hat{y}|$, and $\underline{r}$ is the position vector of the observation point *P*.

Accounting for the considered geometry and the Modified Equivalent Current Approximation (MECA) [11], electric ($\underline{J}_s$) and magnetic ($\underline{M}_s$) PO equivalent surface currents can be used as radiating sources located on the half-plane with PEC backing. They can be so expressed in accordance with (1):

$$\underline{J}_s = \frac{1}{\zeta_0}\left[\left(1-R_\perp\right)E_\perp^i \cos\delta^i\ \hat{u}_\perp + \left(1+R_\parallel\right)E_\parallel^i\left(\hat{y}\times\hat{u}_\perp\right)\right] \cdot \exp\left(jk_0\left(x'\sin\beta'\cos\phi' - z'\cos\beta'\right)\right) \tag{2}$$

$$\underline{M}_s = \left[\left(1-R_\parallel\right)E_\parallel^i \cos\delta^i\ \hat{u}_\perp - \left(1+R_\perp\right)E_\perp^i\left(\hat{y}\times\hat{u}_\perp\right)\right] \cdot \exp\left(jk_0\left(x'\sin\beta'\cos\phi' - z'\cos\beta'\right)\right) \tag{3}$$

The co-ordinates x', z' $(x'>0)$ identify the source point P', ζ_0 is the free-space impedance, $\cos\delta^i = \sin\beta'\sin\phi'$, and the reflection coefficients are determined by using the corresponding ETL models [7], [8]:

$$R_{\parallel,\perp} = \frac{Z_{\parallel,\perp}^{in} - Z_{\parallel,\perp}^0}{Z_{\parallel,\perp}^{in} + Z_{\parallel,\perp}^0} \tag{4}$$

The ETL input impedances $Z_{\parallel,\perp}^{in}$ account for thickness and electromagnetic characteristics of the DNG MTM layer, $Z_\parallel^0 = \zeta_0\cos\delta^i$ and $Z_\perp^0 = \zeta_0/\cos\delta^i$ are the free-space ETL characteristic impedances.

The PO surface current on the PEC half-plane $(x'<0)$ is:

$$\underline{J}_s^{PEC} = \frac{2}{\zeta_0}\left[E_\perp^i \cos\delta^i\ \hat{u}_\perp + E_\parallel^i\left(\hat{y}\times\hat{u}_\perp\right)\right] \cdot \exp\left(jk_0\left(x'\sin\beta'\cos\phi' - z'\cos\beta'\right)\right) \tag{5}$$

3 UAPO Diffracted Field

The UAPO approach allows one to separate the diffracted field related to PO surface currents from the other contributions in the scattered field $\underline{E}^S$.

The surface currents (2), (3) and (5) are now assumed to radiate in the upper half-space $(y>0)$. The corresponding radiation integral provides $\underline{E}^S$, i.e.,

$$\begin{aligned}\underline{E}^S = &-jk_0 \iint_{S_{DNG}} \left(\underline{\underline{I}} - \hat{R}\hat{R}\right)\zeta_0 \underline{J}_s \frac{\exp\left(-jk_0R\right)}{4\pi R} dS + \\ &-jk_0 \iint_{S_{DNG}} \left(\underline{M}_s \times \hat{R}\right) \frac{\exp\left(-jk_0R\right)}{4\pi R} dS + \\ &-jk_0 \iint_{S_{PEC}} \left(\underline{\underline{I}} - \hat{R}\hat{R}\right)\zeta_0 \underline{J}_s^{PEC} \frac{\exp\left(-jk_0R\right)}{4\pi R} dS\end{aligned} \tag{6}$$

where $\underline{\underline{I}}$ is the 3x3 identity matrix, $\hat{R} = (\underline{r}-\underline{r}')/R$ and $R = |\underline{r}-\underline{r}'|$, $\underline{r}'$ being the position vector of P'.

If P' is on the edge, $\hat{R} \cong \hat{s} = \sin\beta'\cos\phi\,\hat{x} + \sin\beta'\sin\phi\,\hat{y} + \cos\beta'\,\hat{z}$ can be considered in (6). The following matrix representation is permitted by such an approximation with the use of convenient local co-ordinate systems as in [4]:

$$\underline{E}^S = \begin{pmatrix} E_\beta^s \\ E_\phi^s \end{pmatrix} \cong \left[I_{DNG}^s \underline{\underline{A}} + I_{PEC}^s \underline{\underline{B}}\right] \begin{pmatrix} E_{\beta'}^i \\ E_{\phi'}^i \end{pmatrix} \tag{7}$$

The involved matrices are:

$$\underline{\underline{A}} = \underline{\underline{A}}_1\left[\underline{\underline{A}}_2\underline{\underline{A}}_4\underline{\underline{A}}_5 + \underline{\underline{A}}_3\underline{\underline{A}}_4\underline{\underline{A}}_6\right]\underline{\underline{A}}_7 \tag{8}$$

$$\underline{\underline{B}} = \underline{\underline{A}}_1\left[\underline{\underline{B}}_2\underline{\underline{B}}_4\underline{\underline{B}}_5\right]\underline{\underline{A}}_7 \tag{9}$$

with

$$\underline{\underline{A}}_1 = \begin{pmatrix} \cos\beta'\cos\phi & \cos\beta'\sin\phi & -\sin\beta' \\ -\sin\phi & \cos\phi & 0 \end{pmatrix} \tag{10}$$

$$\underline{\underline{A}}_2 = \begin{pmatrix} 1-\sin^2\beta'\cos^2\phi & -\sin\beta'\cos\beta'\cos\phi \\ -\sin^2\beta'\sin\phi\cos\phi & -\sin\beta'\cos\beta'\sin\phi \\ -\sin\beta'\cos\beta'\cos\phi & \sin^2\beta' \end{pmatrix} \tag{11}$$

$$\underline{\underline{A}}_3 = \begin{pmatrix} 0 & -\sin\beta'\sin\phi \\ -\cos\beta' & \sin\beta'\cos\phi \\ \sin\beta'\sin\phi & 0 \end{pmatrix} \tag{12}$$

$$\underline{\underline{A}}_4 = \frac{1}{G(\beta',\phi')}\begin{pmatrix} -\cos\beta' & -\sin\beta'\cos\phi' \\ -\sin\beta'\cos\phi' & \cos\beta' \end{pmatrix} \tag{13}$$

$$\underline{\underline{A}}_5 = \begin{pmatrix} 0 & (1-R_\perp)\sin\beta'\sin\phi' \\ 1+R_\| & 0 \end{pmatrix} \tag{14}$$

$$\underline{\underline{A}}_6 = \begin{pmatrix} (1-R_\|)\sin\beta'\sin\phi' & 0 \\ 0 & -1-R_\perp \end{pmatrix} \tag{15}$$

$$\underline{\underline{A}}_7 = \frac{1}{G(\beta',\phi')}\begin{pmatrix} \cos\beta'\sin\phi' & \cos\phi' \\ -\cos\phi' & \cos\beta'\sin\phi' \end{pmatrix} \tag{16}$$

$$\underline{\underline{B}}_2 = \begin{pmatrix} 1-\sin^2\beta'\cos^2\phi & -\sin^2\beta'\sin\phi\cos\phi & -\cos\beta'\sin\beta'\cos\phi \\ -\sin^2\beta'\sin\phi\cos\phi & 1-\sin^2\beta'\sin^2\phi & -\cos\beta'\sin\beta'\sin\phi \\ -\cos\beta'\sin\beta'\cos\phi & -\cos\beta'\sin\beta'\sin\phi & \sin^2\beta' \end{pmatrix} \tag{17}$$

$$\underline{\underline{B}}_4 = \frac{1}{G(\beta',\phi')}\begin{pmatrix} -\cos\beta' & -\sin\beta'\cos\phi' \\ 0 & 0 \\ -\sin\beta'\cos\phi' & \cos\beta' \end{pmatrix} \tag{18}$$

$$\underline{\underline{B}}_5 = \begin{pmatrix} 0 & 2\sin\beta'\sin\phi' \\ 2 & 0 \end{pmatrix} \tag{19}$$

wherein $G(\beta',\phi') = \sqrt{1-\sin^2\beta'\sin^2\phi'}$.

According to [8], [9], the integrals $I^s_{DNG,PEC}$ in (7) lead to the corresponding diffraction terms $I^d_{DNG,PEC}$:

$$I^d_{DNG} = \frac{\exp(-j\pi/4)}{2\sqrt{2\pi k_0}}\frac{\exp(-jk_0 s)}{\sqrt{s}} \cdot \frac{F_t\left(2k_0 s\sin^2\beta'\cos^2\left(\frac{\phi+\phi'}{2}\right)\right)}{\sin^2\beta'(\cos\phi+\cos\phi')} \tag{20}$$

$$I^d_{PEC} = -\frac{\exp(-j\pi/4)}{2\sqrt{2\pi k_0}}\frac{\exp(-jk_0 s)}{\sqrt{s}} \cdot \frac{F_t\left(2k_0 s\sin^2\beta'\cos^2\left(\frac{(\pi-\phi)+(\pi-\phi')}{2}\right)\right)}{\sin^2\beta'(\cos\phi+\cos\phi')} \tag{21}$$

where $F_t(\cdot)$ symbolizes the UTD transition function [4] and *s* is the distance from the diffraction point to *P*.

The above terms lead to the following formulation of the UAPO diffracted field in the UTD context:

$$\begin{pmatrix} E^d_\beta \\ E^d_\phi \end{pmatrix} = \left[I^d_{DNG}\underline{\underline{A}} + I^d_{PEC}\underline{\underline{B}} \right]\begin{pmatrix} E^i_{\beta'} \\ E^i_{\phi'} \end{pmatrix} = = \underline{\underline{D}}\,\frac{\exp(-jk_0 s)}{\sqrt{s}}\begin{pmatrix} E^i_{\beta'} \\ E^i_{\phi'} \end{pmatrix} \tag{22}$$

4 Numerical Examples

Numerical tests have been performed to prove the efficiency of the UAPO diffracted field to compensate the GO field at the reflection boundary. The half-circumference with $0 < \phi < 180°$ and radius equal to $5\lambda_0$, λ_0 being the free-space wavelength, has been chosen as observation domain in the reported examples (see from Fig. 2 to Fig. 5). These last refer to a DNG MTM half-screen with relative permittivity $\varepsilon_r = -6.5 - j0.0013$, relative permeability $\mu_r = -1$, and thickness $d = 0.1\lambda_0$. Two values of ϕ' have been selected to obtain reflection boundaries in the second (see Figs. 2 and 3) and first (see Figs. 4 and 5) quadrants.

The total field graphs highlight unbroken behaviors at the reflection boundaries related to various incidence directions, thus demonstrating the aptitude of the UAPO diffracted field to solve the GO field problems.

5 Conclusions

Numerical tests have proved the ability of the proposed UAPO solution to supply field values able to compensate the GO field discontinuities at the considered reflection boundaries. A full-wave numerical tool will be used in a forthcoming paper to test the solution accuracy. It must be stressed that the proposed solution is presented in the UTD matrix form and is easy to handle.

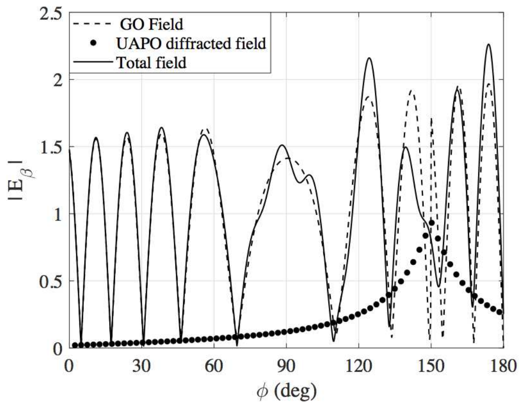

Figure 2. β-components when $E^i_{\beta'} = 1$, $E^i_{\phi'} = 0$ and $\beta' = 70°$, $\phi' = 30°$.

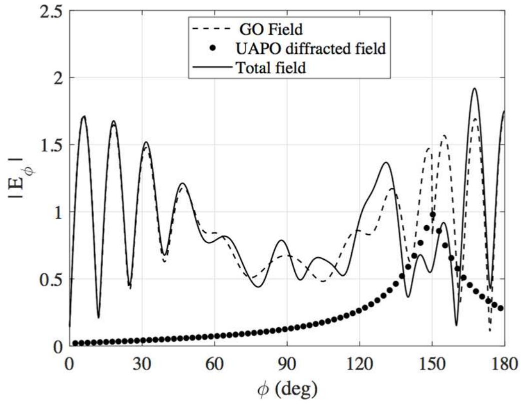

Figure 3. ϕ-components when $E^{i}_{\beta'} = 0$, $E^{i}_{\phi'} = 1$ and $\beta' = 70°,\ \phi' = 30°$.

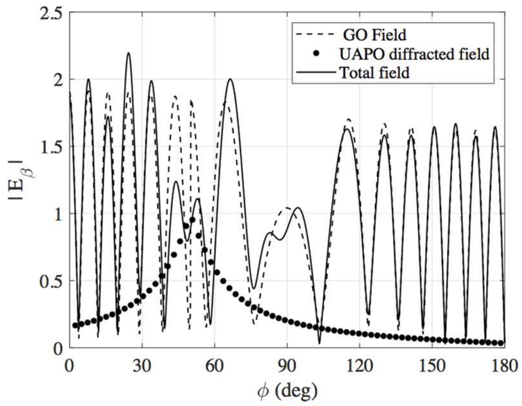

Figure 4. β-components when $E^{i}_{\beta'} = 1$, $E^{i}_{\phi'} = 0$ and $\beta' = 70°,\ \phi' = 130°$.

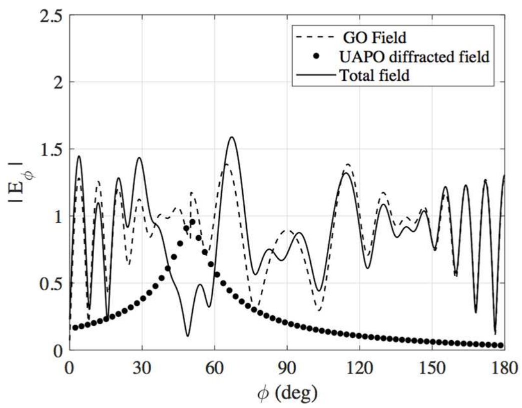

Figure 5. ϕ-components when $E^{i}_{\beta'} = 0$, $E^{i}_{\phi'} = 1$ and $\beta' = 70°,\ \phi' = 130°$.

6 References

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