URSI GASS 2021, Rome, Italy, 28 August - 4 September 2021

UNIFORM ASYMPTOTIC SOLUTION FOR DIELECTRIC WEDGE DIFFRACTION BASED ON EQUIVALENT CURRENTS

Duc Minh Nguyen,[(1)] Hiroshi Shirai[(1)] and Se-Yun Kim[(2)]
(1) Graduate School of Science and Engineering, Chuo University, Tokyo, Japan
(2) Center for Imaging Media Research, Korea Institute of Science and Technology, Seoul, Korea

Abstract

In this study, a uniform asymptotic solution is proposed to calculate the diffraction field by a dielectric wedge based on the equivalent current method. In this formulation, the scattering field is obtained from the radiation integral of the equivalent electric and magnetic currents determined by the geometrical optics (GO) field on the wedge surface. Numerical calculations are made to compare the results with those by other methods, such as the conventional physical optics approximation (PO) and the hidden rays of diffraction (HRD). A good agreement has been observed to confirm the validity of our method.

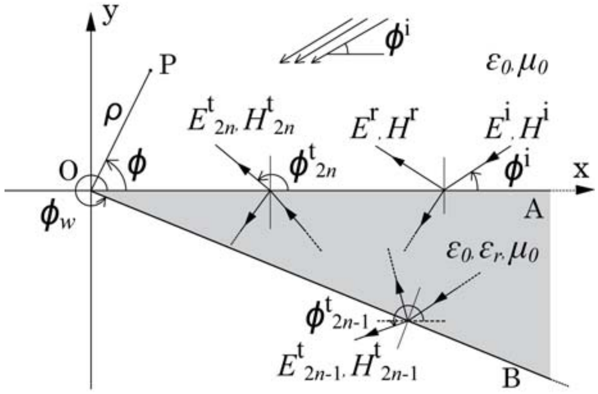

Figure 1. Dielectric wedge.

1 Introduction

Electromagnetic diffraction by a wedge is one of the fundamental but important problems [1]. While the scattering fields by large electric conducting objects may be solved based on the physical optics (PO) method [2, 3], those by dielectric objects may be analyzed as a radiation from the equivalent electric and magnetic currents on a postulated surface. In this paper, a uniform asymptotic solution of the dielectric wedge diffraction is derived from the equivalent currents which are obtained from the geometrical optics (GO) field on the wedge surface. In the following discussion, the time-harmonic factor $e^{j\omega t}$ is assumed and suppressed throughout the text.

2 Formulation

As shown in Fig. 1, let us consider a two-dimensional dielectric wedge of the wedge angle ϕ_w and the dielectric constant ε_r is illuminated by an E polarized plane wave:

$$E_z^{\mathrm{i}} = e^{jkx\cos\phi^{\mathrm{i}} + jky\sin\phi^{\mathrm{i}}}. \qquad (1)$$

Here $k = \omega\sqrt{\varepsilon_0\mu_0}$ denotes the free space wave number. For simplicity, let us assume that the incident plane wave illuminates surface OA only ($0 < \phi^{\mathrm{i}} < \phi_w - \pi$).

According to the surface equivalence theorem, the scattering fields can be considered as radiation from equivalent electric and magnetic currents on a virtual surface enclosing the scattering body. For the large objects, the equivalent currents may be approximated in terms of GO ray fields which emit from this virtual surface pointing outward.

When the incident wave impinges on the illuminated surfaces, it excites the reflected and transmitted waves. In addition, the primary transmitted wave continues to experience the internal reflection and emits the outgoing successive transmitted waves ($\boldsymbol{E}_n^{\mathrm{t}}$, $\boldsymbol{H}_n^{\mathrm{t}}$). Therefore, the scattering field on the illuminated surface OA is given by the reflected wave ($\boldsymbol{E}^{\mathrm{r}}$, $\boldsymbol{H}^{\mathrm{r}}$) and the $2n$-th transmitted waves ($\boldsymbol{E}_{2n}^{\mathrm{t}}$, $\boldsymbol{H}_{2n}^{\mathrm{t}}$). Accordingly, one can obtain the corresponding equivalent electric and magnetic currents $\boldsymbol{J}^{\mathrm{r}}$, $\boldsymbol{M}^{\mathrm{r}}$, $\boldsymbol{J}_{2n}^{\mathrm{t}}$ and $\boldsymbol{M}_{2n}^{\mathrm{t}}$. On other hand, the scattering field emanating from the shadow surface OB may be approximated by the negative incident wave ($-\boldsymbol{E}^{\mathrm{i}}$, $-\boldsymbol{H}^{\mathrm{i}}$) to cancel the original incident field, and transmitted waves ($\boldsymbol{E}_{2n-1}^{\mathrm{t}}$, $\boldsymbol{H}_{2n-1}^{\mathrm{t}}$). Thus, the equivalent currents $\boldsymbol{J}^{\mathrm{i}}$, $\boldsymbol{M}^{\mathrm{i}}$, $\boldsymbol{J}_{2n-1}^{\mathrm{t}}$ and $\boldsymbol{M}_{2n-1}^{\mathrm{t}}$ on the shadowed surface OB may be derived from them. Then, the scattering fields due to above currents can be calculated by integrating using two-dimensional free space Green's function. These radiation integrals can be evaluated by the saddle point technique to get the uniform asymptotic solution.

For example, z-component of the scattering field $\hat{\boldsymbol{E}}_{2n}^{\mathrm{t}}$ given by the $2n$-th transmitted waves on the surface OA can be derived from the corresponding currents $\boldsymbol{J}_{2n}^{\mathrm{t}}$ and $\boldsymbol{M}_{2n}^{\mathrm{t}}$ as:

$$\begin{aligned}
\{\hat{\boldsymbol{E}}_{2n}^{\mathrm{t}}\}_z = &-\frac{T_{2n}}{\sqrt{\pi}} e^{jk\rho\cos(\pi-\phi_{2n}^{\mathrm{t}}+|\phi|)}\,\mathrm{sgn}(\phi_{2n}^{\mathrm{t}}-|\phi|) \\
&\cdot Q\left[(1+j)\left|\cos\left\{\frac{\pi-\phi_{2n}^{\mathrm{t}}+|\phi|}{2}\right\}\right|\sqrt{k\rho}\right]U(\phi) \\
&-T_{2n}C(k\rho)\left(\frac{\sin\phi_{2n}^{\mathrm{t}}+\sin\phi}{\cos\phi-\cos\phi_{2n}^{\mathrm{t}}}\right. \\
&\left.-\frac{1}{\cos\{(\pi-\phi_{2n}^{\mathrm{t}}+|\phi|)/2\}}U(\phi)\right) \\
&+T_{2n}e^{jk\rho\cos(\pi-\phi_{2n}^{\mathrm{t}}+|\phi|)}U(\phi_{2n}^{\mathrm{t}}-|\phi|)U(\phi), \qquad (2)
\end{aligned}$$

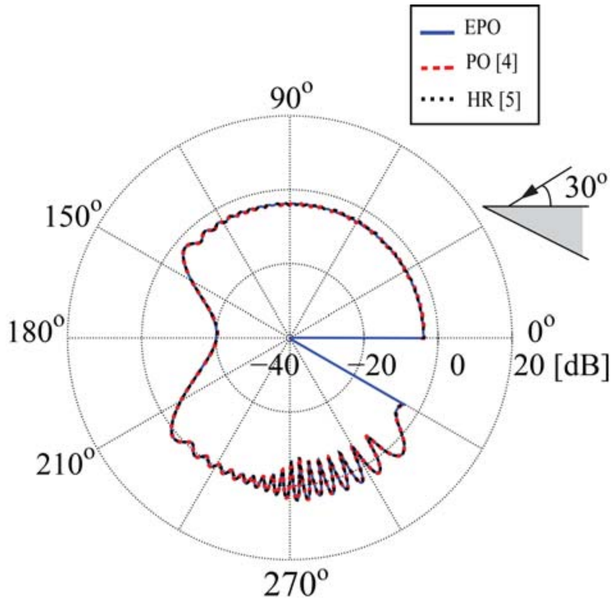

Figure 2. Total scattering field outside the dielectric wedge: $\phi_w = 330°$, $\phi^{\mathrm{i}} = 30°$, $\varepsilon_r = 6$ and $\rho = 20\lambda$.

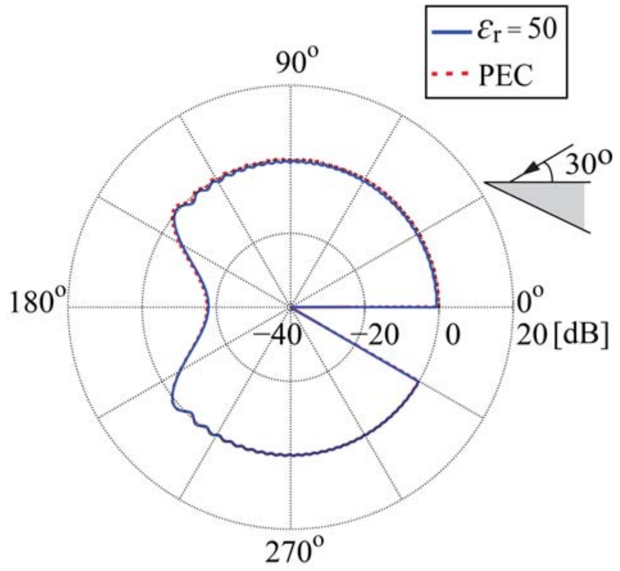

Figure 3. Total scattering field outside the wedge for the dielectric constant $\varepsilon_r = 50$ and the PEC cases: $\phi_w = 330°$, $\phi^{\mathrm{i}} = 30°$ and $\rho = 20\lambda$.

where

$$C(\chi) = (8\pi\chi)^{-1/2} e^{-j(\chi+\pi/4)}, \tag{3}$$

and

$$Q(x) = \int_x^{\infty} \exp(-y^2)dy \tag{4}$$

is complementary error function. $U(x)$ and $\mathrm{sgn}(x)$ are unit step and sign functions, respectively. Cumulative transmission coefficient T_{2n} in Eq. (2) can be defined as:

$$T_{2n} = (1+\Gamma_0)(1+\Gamma_{2n})\Gamma_{2n-1}...\Gamma_2\Gamma_1, \tag{5}$$

where primary reflection coefficient Γ_0 and $2n$-th internal reflection coefficient Γ_{2n} are given as:

$$\Gamma_0 = \frac{\sin\phi^{\mathrm{i}} - \sqrt{\varepsilon_r - \cos^2\phi^{\mathrm{i}}}}{\sin\phi^{\mathrm{i}} + \sqrt{\varepsilon_r - \cos^2\phi^{\mathrm{i}}}}, \tag{6}$$

$$\Gamma_{2n} = \frac{\sqrt{\varepsilon_r}\sin\phi^{\mathrm{i}}_{2n} - \sqrt{1-\varepsilon_r\cos^2\phi^{\mathrm{i}}_{2n}}}{\sqrt{\varepsilon_r}\sin\phi^{\mathrm{i}}_{2n} + \sqrt{1-\varepsilon_r\cos^2\phi^{\mathrm{i}}_{2n}}}. \tag{7}$$

Similarly, one can obtain the scattering fields: $\hat{\boldsymbol{E}}^{\mathrm{r}}$ due to the original reflected wave, $\hat{\boldsymbol{E}}^{\mathrm{i}}$ due to the negative incident wave, and $\hat{\boldsymbol{E}}^{\mathrm{t}}_{2n-1}$ due to the $(2n-1)$-th transmitted wave. Finally, the total scattering field $\hat{\boldsymbol{E}}^{\mathrm{s}}$ outside the dielectric wedge can be found by summing up the above contributions, if any, as:

$$\hat{\boldsymbol{E}}^{\mathrm{s}} = \hat{\boldsymbol{E}}^{\mathrm{r}} + \hat{\boldsymbol{E}}^{\mathrm{i}} + \sum_{n=1}^{N} \hat{\boldsymbol{E}}^{\mathrm{t}}_n, \tag{8}$$

where N is the maximum number of possible internal reflections before the total reflection.

3 Numerical results and discussion

Figure 2 shows the total scattering pattern calculated for $\rho = 20\lambda$ and $\phi^{\mathrm{i}} = 30°$. Our result named as EPO (extended physical optics) is compared with those obtained from the PO [4] and the hidden rays of diffraction (HRD) [5] methods. In this calculation, there is only one outgoing transmitted wave from the surface OB, and this contribution yields a oscillatory pattern in the shadowed region. Our result coincides with those by PO and HRD for this setting.

Figure 3 shows the scattering pattern for highly refractive case ($\varepsilon_r = 50$), which can be well compared with the case of PEC surface. This agreement demonstrates the validity of our method.

4 Acknowledgements

A part of this work is supported by the Scientific Research Grant-In-Aid (19K04398, 2019) from JSPS (Japan Society for the Promotion of Science), Japan.

References

[1] J. J. Bowman, T. B. A. Senior and P. L. E. Uslenghi, *Electromagnetic and Acoustic Scattering by Simple Shapes,* Amsterdam: North-Holland, 1969.

[2] H. Shirai, *Geometrical Theory of Diffraction*, Corona Publ. Co., 2015 (in Japanese).

[3] H. N. Quang and H. Shirai, "A new interpretation of physical optics approximation from surface equivalence theorem," *IEICE Trans. Electron.*, vol. E101-C, no. 8, pp. 664–670, 2018.

[4] S. Y. Kim, J. W. Ra, and S. Y. Shin, "Diffraction by an arbitrary-angled dielectric wedge-Part I; Physical optics approximation," *IEEE Trans. Antennas Propaga.*, vol. AP-39, no. 9, pp. 1272–1281, 1991.

[5] S. Y. Kim, "Hidden rays of diffraction," *IEEE Trans. Antennas Propaga.*, vol. AP-55, no. 3, pp. 892–906, 2007.