The UAPO Solution for the Plane Wave Diffraction by a Resistive Half-Plane in the Case of Skew Incidence

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Abstract

The extension of the Uniform Asymptotic Physical Optics approach to the three-dimensional diffraction problem involving a resistive half-plane is presented in this work. The resistive boundary conditions in correspondence of the screen are used to determine the reflection and transmission coefficients for both the polarizations and then the Geometrical Optics field around the structure. This enables the formulation of the electric and magnetic equivalent surface currents to be considered as radiating sources in the approach for the evaluation of the diffracted field. The ability of this contribution to compensate the jumps of the Geometrical Optics field is proved by means of numerical tests.

1 Introduction

The resistive boundary conditions define the performance of the electromagnetic field in correspondence of thin lossy dielectric sheets by means of the surface resistivity R_e and imply the existence of an electric surface current whose strength is proportional to the common value of the tangential electric field at the surface [1]. The sheet is perfectly conducting if R_e is equal to zero. The resistive boundary conditions permit to model many simple structures and analytical methods have been proposed in literature to solve related diffraction problems arising from the interaction of plane waves with truncated planar screens.

The Uniform Asymptotic Physical Optics (UAPO) approach has been presented in [2] to solve the plane wave diffraction by a junction of equal resistive sheets having a non-planar configuration. In particular, plane waves that are characterized by normal incidence with respect to the junction and by incident electric field parallel to the junction have been considered in a two-dimensional propagation model. Comparisons with data resulting from a numerical technique based on the Boundary Element Method (BEM) have assessed the accuracy of the proposed approach to solve the investigated scalar problem. Unfortunately, two-dimensional propagation models have limited applications in real scenarios. Accordingly, the extension of the UAPO approach to plane waves with arbitrarily polarized electric fields at skew incidence with respect to junctions of resistive sheets is worthy to be investigated. Accounting for the peculiarities of the UAPO approach (see [2]-[12] for a non-exhaustive list of references), the first step needs to study the plane wave diffraction by an isolated resistive half-plane.

This work deals with the plane wave diffraction by a resistive half-plane in the case of an arbitrarily polarized electric field at skew incidence with respect to the edge. The corresponding UAPO solution is given in the context of the Uniform Geometrical Theory of Diffraction (UTD) [13] by taking into account the local reference systems in Fig. 1. The resistive boundary conditions in correspondence of an infinite screen are used to determine the reflection and transmission coefficients for the electric field components parallel and perpendicular to the ordinary plane of incidence. It must be stressed that the knowledge of the reflection and transmission coefficients is necessary not only for the computation of the Geometrical Optics (GO) field in the surrounding free space, but also for the evaluation of the electric and magnetic PO equivalent surface currents in the radiation integral at the basis of the UAPO approach, which allows one to isolate the high-frequency diffraction contribution. Obviously, only electric radiating source exists for the resistive half-plane.

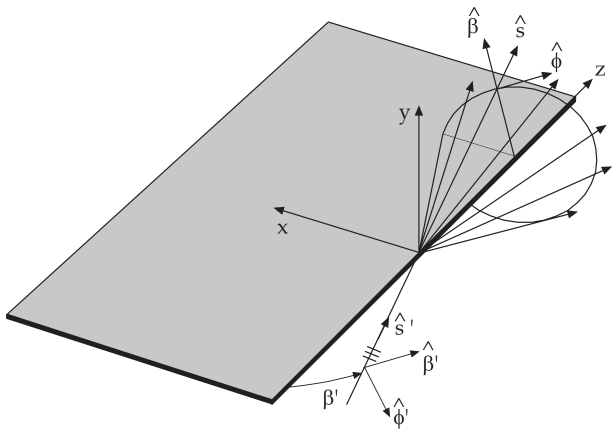

Figure 1. Plane wave diffraction by a resistive half-plane.

2 PO Radiating Sources

Let us consider a resistive screen S lying in the plane $y=0$ and interacting with an incident electric field given by:

$$\underline{E}^i = \left[E_\perp^i \, \hat{u}_\perp + E_\parallel^i \left(\hat{u}_\perp \times \hat{s}' \right) \right] \exp\left(-jk_0 \hat{s}' \cdot \underline{r} \right) \qquad (1)$$

where k_0 is the free-space propagation constant, $\hat{s}' = -\sin\beta' \cos\phi' \, \hat{x} - \sin\beta' \sin\phi' \, \hat{y} + \cos\beta' \, \hat{z}$ is the unit vector of the incidence direction, $\hat{u}_\perp = (\hat{s}' \times \hat{y}) / \left| \hat{s}' \times \hat{y} \right|$, and $\underline{r}$ denotes the position vector of the observation point P. The parallel (||) and perpendicular (⊥) field components are used in (1).

The electromagnetic field on S satisfies the following boundary conditions [1]:

$$\hat{y} \times \left[\underline{E}^+ - \underline{E}^- \right] \Big|_S = 0 \qquad (2)$$

$$\hat{y} \times \left[\hat{y} \times \underline{E} \right] \Big|_S = -R_e \, \hat{y} \times \left[\underline{H}^+ - \underline{H}^- \right] \Big|_S \qquad (3)$$

wherein the superscripts + and – identifies the field on the upper and lower parts of S. The surface resistance R_e is:

$$R_e = -\frac{j\zeta_0}{k_0 d (\varepsilon_r - 1)} \qquad (4)$$

where d and ε_r are the thickness and the relative permittivity of the screen, respectively, and ζ_0 is the free-space impedance.

Accounting for (1) and (2), the electric ($\underline{J}_s$) and magnetic ($\underline{J}_{ms}$) PO equivalent surface currents to be used in the radiation integral of the UAPO approach can be so expressed:

$$\underline{J}_s = \hat{y} \times \left[\underline{H}^+ - \underline{H}^- \right] \Big|_S = \frac{1}{\zeta_0} \Big[\left(1 - R_\perp - T_\perp \right) E_\perp^i \cos\theta^i \, \hat{u}_\perp + \left(1 + R_\parallel - T_\parallel \right) E_\parallel^i \left(\hat{y} \times \hat{u}_\perp \right) \Big] \exp\left(-jk_0 \hat{s}' \cdot \underline{r}' \right) \qquad (5)$$

$$\underline{J}_{ms} = \left[\underline{E}^+ - \underline{E}^- \right] \Big|_S \times \hat{y} = 0 \qquad (6)$$

Formula (5) contains the position vector $\underline{r}'$ on S, the standard angle of incidence θ^i, and the reflection (R) and transmission (T) coefficients for both the polarizations. These last are determined by applying the boundary conditions (2) and (3):

$$R_\parallel = \frac{\cos\theta^i}{\gamma + \cos\theta^i} \qquad (7)$$

$$T_\parallel = \frac{\gamma}{\gamma + \cos\theta^i} \qquad (8)$$

$$R_\perp = -\frac{1}{1 + \gamma \cos\theta^i} \qquad (9)$$

$$T_\perp = \frac{\gamma \cos\theta^i}{1 + \gamma \cos\theta^i} \qquad (10)$$

wherein $\gamma = 2R_e / \zeta_0$.

3 UAPO Diffraction by a Resistive Half-Plane

The electric surface current density (5) can be used as source in the PO radiation integral for evaluating the scattered field $\underline{E}^S$ due to a resistive half-plane S_{hp} in the xz-plane with $x > 0$ (see Fig. 1), i.e.,

$$\underline{E}^S \cong -jk_0 \iint_{S_{hp}} \left(\underline{\underline{I}} - \hat{R}\hat{R} \right) \zeta_0 \underline{J}_s \frac{\exp\left(-jk_0 R \right)}{4\pi R} \mathrm{d}S_{hp} \qquad (11)$$

where $\hat{R} = (\underline{r} - \underline{r}')/R$, $R = \left| \underline{r} - \underline{r}' \right|$ and $\underline{\underline{I}}$ is the 3x3 identity matrix.

The approximation $\hat{R} \cong \hat{s}$ ($\hat{s}$ is the unit vector of the diffraction direction on the Keller's cone) is then accepted, thus leading to the following matrix representation:

$$\underline{E}^S = \begin{pmatrix} E_\beta^s \\ E_\phi^s \end{pmatrix} \cong I^S \, \underline{\underline{M}} \begin{pmatrix} E_{\beta'}^i \\ E_{\phi'}^i \end{pmatrix} \qquad (12)$$

where the scattered field is related to the incident one accounting for useful local co-ordinate systems as in [13].

The matrix $\underline{\underline{M}}$ in (12) is given by:

$$\underline{\underline{M}} = \underline{\underline{M}}_1 \underline{\underline{M}}_2 \underline{\underline{M}}_3 \underline{\underline{M}}_4 \underline{\underline{M}}_5 \qquad (13)$$

with

$$\underline{\underline{M}}_1 = \begin{pmatrix} \cos\beta'\cos\phi & \cos\beta'\sin\phi & -\sin\beta' \\ -\sin\phi & \cos\phi & 0 \end{pmatrix} \quad (14)$$

$$\underline{\underline{M}}_2 = \begin{pmatrix} 1-\sin^2\beta'\cos^2\phi & -\sin\beta'\cos\beta'\cos\phi \\ -\sin^2\beta'\sin\phi\cos\phi & -\sin\beta'\cos\beta'\sin\phi \\ -\sin\beta'\cos\beta'\cos\phi & \sin^2\beta' \end{pmatrix} \quad (15)$$

$$\underline{\underline{M}}_3 = \frac{1}{A(\beta',\phi')}\begin{pmatrix} -\cos\beta' & -\sin\beta'\cos\phi' \\ -\sin\beta'\cos\phi' & \cos\beta' \end{pmatrix} \quad (16)$$

$$\underline{\underline{M}}_4 = \begin{pmatrix} 0 & (1-R_\perp - T_\perp)\sin\beta'\sin\phi' \\ 1+R_\| - T_\| & 0 \end{pmatrix} \quad (17)$$

$$\underline{\underline{M}}_5 = \frac{1}{A(\beta',\phi')}\begin{pmatrix} \cos\beta'\sin\phi' & \cos\phi' \\ -\cos\phi' & \cos\beta'\sin\phi' \end{pmatrix} \quad (18)$$

wherein $A(\beta',\phi') = \sqrt{1-\sin^2\beta'\sin^2\phi'}$.

The UAPO approach allows one to isolate the high-frequency diffraction contribution from the evaluation of I^s in (12). Accounting for integral evaluations and integral representations involving the zeroth order Hankel function of second kind, it is possible to apply the steepest descent method and the multiplicative method in the high-frequency approximation, thus obtaining the UAPO diffraction term I^d:

$$I^d = \frac{\exp(-j\pi/4)}{2\sqrt{2\pi k_0}} \frac{\exp(-jk_0 s)}{\sqrt{s}} \cdot \frac{F_t\left(2k_0 s \sin^2\beta'\cos^2\left(\frac{\phi\pm\phi'}{2}\right)\right)}{\sin^2\beta'(\cos\phi+\cos\phi')} \quad (19)$$

The sign + (–) must be used if $0<\phi<\pi$ $(\pi<\phi<2\pi)$.

The above term permits to formulate the UAPO diffracted field in the UTD framework as follows:

$$\begin{pmatrix} E^d_\beta \\ E^d_\phi \end{pmatrix} = I^d \underline{\underline{M}} \begin{pmatrix} E^i_{\beta'} \\ E^i_{\phi'} \end{pmatrix} = \underline{\underline{D}} \frac{\exp(-jk_0 s)}{\sqrt{s}} \begin{pmatrix} E^i_{\beta'} \\ E^i_{\phi'} \end{pmatrix} \quad (20)$$

where s is the distance from the diffraction point to P. Accordingly, the diffraction matrix $\underline{\underline{D}}$ is given by:

$$\underline{\underline{D}} = \sqrt{s}\exp(jk_0 s)\, I^d\, \underline{\underline{M}} \quad (21)$$

4 Numerical Tests

The following figures refer to a resistive half-plane having $\varepsilon_r = 2.94 - j0.033$, $\mu_r = 1$ and $d = 0.1\lambda_0$, where λ_0 is the free-space wavelength. The observation domain is a circular path with radius equal to $5\lambda_0$ and the incidence directions are chosen such that ϕ' belongs to the first and second quadrants.

As displayed in Figs. 2-5, the magnitude of the GO field possesses jumps at the reflection and transmission boundaries, whereas the UAPO diffracted field shows its maxima in correspondence of these directions as expected. The continuity of the total field in the complete observation range assesses the ability of the UAPO solution to counterbalance the GO field discontinuities.

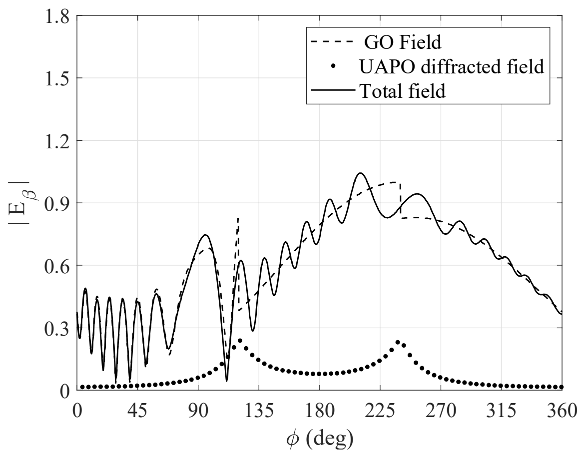

Figure 2. β-components when $E^i_{\beta'} = 1$, $E^i_{\phi'} = 0$ and $\beta' = 50°, \phi' = 60°$.

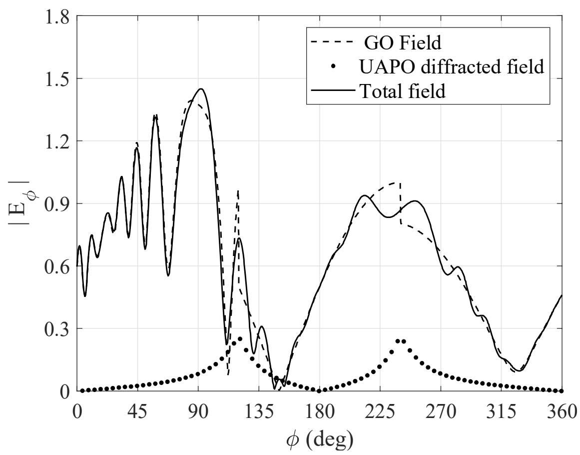

Figure 3. ϕ-components when $E^i_{\beta'} = 0$, $E^i_{\phi'} = 1$ and $\beta' = 50°, \phi' = 60°$.

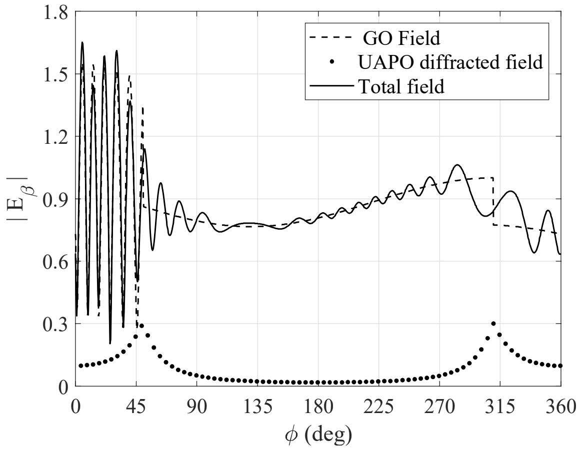

Figure 4. β-components when $E^i_{\beta'} = 1$, $E^i_{\phi'} = 0$ and $\beta' = 70°, \phi' = 130°$.

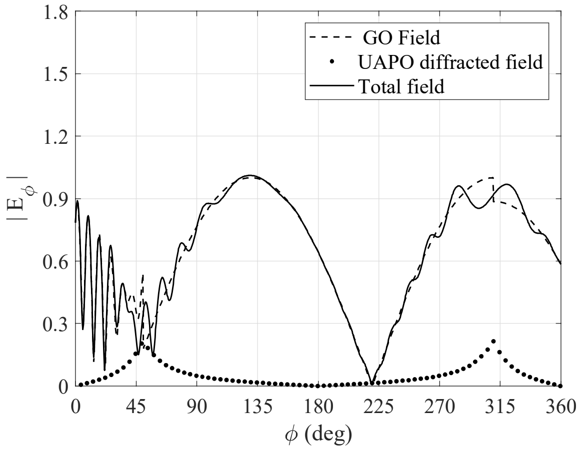

Figure 5. ϕ-components when $E^i_{\beta'} = 0$, $E^i_{\phi'} = 1$ and $\beta' = 70°, \phi' = 130°$.

5 Concluding Remarks

The UAPO approach has been conveniently exploited to tackle the plane wave diffraction by a resistive half-screen when the incidence direction is oblique with respect to the edge. The resulting solution has been presented in the UTD matrix form and its ability to compensate the jumps of the GO field at the shadow boundaries has been proved. The accuracy of the proposed UAPO solution will be tested in future works by using a full-wave numerical tool.

6 References

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