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Compressibility of Generalized Integral Equation Formulations

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Recent years have seen great advances in the development of fast direct integral-equation solvers, for electromagnetic problems. Various algorithms were proposed for the structured compression of the impedance matrix and computation of its compressed inverse. These can be classified into two main categories: (i) algorithms that use the rank-deficiency of field-integral interactions for the straightforward low-rank (single level) approximation the impedance matrix blocks and (ii) algorithms that compute these blocks' butterfly (multilevel) approximations, which rely on the butterfly-rank remaining roughly fixed when observer- and source-subdomains are partitioned and combined, respectively [1]. The latter, more complicated for implementation, class has been shown to achieve superior asymptotic scaling of the compression, for arbitrary and large compared to the wavelength topologies. The former class is only effective for geometrical configurations and integral equation kernels that yield inherently very rank-deficient interactions. Such are the cases of elongate and quasi-planar wave problems, where the rank increases, with the problem's dimensions, slower than the number of problem unknowns [2]. With the aim of extending the range of problems that can be efficiently solved via low-rank compression, generalized source integral equations were proposed [3]. These reduce the problem's effective dimensionality and, thus, increase its low-rank compressibility. It was recently shown that the enhanced rank-deficiency of generalized source integral equation interactions also leads to enhanced butterfly compressibility of the corresponding matrix blocks [4]. However, this advantage diminishes for sufficiently low compression thresholds; While the strong reduced-dimensionality line-of-sight component of the interaction remains rank deficient, the rank associated with its weaker diffractive portion increases fast with the participating clusters' sizes.

In this work, we will explore strategies for improving the compression of matrix blocks that result from the generalized source integral equations, especially for lower compression thresholds. Among the examined ideas will be the combination of butterfly approximations of varying depth for compressed representation of different components of the blocks' spectrum. Particular attention will be given to the case of using a (single-level) low-rank approximation for representing only the line-of-sight component of an interaction and a multilevel representation for representing its remainder, to a prescribed accuracy. Examples will include various cases of source-observer subdomain configurations that correspond to different types of algebraic solution schemes/admissibility criteria. For the various cases, it will be examined whether and how an initial assessment of the singular values spectrum of impedance matrix blocks can be used to inform the choice of parameters for optimizing the compression.

References

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