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Application of subdivision surface and Laplace-Beltrami Operators to Integral Equations

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The Laplace-Beltrami operator (LBO) offers the means to smoothly interpolate data on a surface. As a result, they have been used for a number of applications ranging from solutions to physical problems such as the heat equation to fluid dynamics to electrostatics and so on. In addition, it has been widely used in computational geometry and more recently, data science. Of interest to us is application in electromagnetics. Here the literature is considerably sparser. The most immediately relevant application is the representation/manipulation of Debye sources on a manifold. But, to better set the stage for the discussion, note that the LBO is a self-adjoint linear differential equation defined on a manifold. The eigenfunctions of these operators can be used to form a "spectral" decomposition of the function defined on a manifold. For instance, the eigenfunctions of LBO on the surface of a sphere are spherical harmonics. As a result, these form what is known as manifold harmonics. In addition to being a basis set that can be used to represent physical quantities, they provide a compact, elegant, and multi-resolution basis for spectral *shape* processing. Indeed, it can be shown that very complex shapes can be represented to sufficiently high fidelity using relatively few eigenfunctions.

In this paper, our goals are to examine the various uses of manifold harmonics for applications to problems in both acoustics and electromagnetics. The features that we seek to exploit are the natural compression that these eigenfunctions provide to concoct an algorithm for shape optimization. To this end, the foundation of our approach will depend on subdivision surface representation of surfaces. This representation provides a description of a surface that is C^2 almost everywhere. Using subdivision geometric basis sets permit constructing a variational system to help with obtaining eigenfunctions on a geometry. These basis, in turn, can be used to represent both the physics and the geometry. We will take advantage of this approach to demonstrate (a) compressible representation of geometry, (b) compressible representation of the physical system, (c) shape optimization/inverse source reconstruction using LBO representations, and (d) potential extension to electromagnetics.