Eigenfrequency Spectrum of Prolate Spheroidal Magneto-optic Cavities

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Outline

1. Introduction
2. Solution of the problem
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Aims of the study

- Investigation of the eigenfrequencies of prolate spheroidal magneto-optic cavities
- Scattering formulation employing spheroidal eigenvectors for the expansion of the incident, interior and scattered fields
- Application of a root-finding algorithm for obtaining the eigenfrequencies

1 Introduction

Magneto-optic cavities

- Magneto-optical coupling between spin and electromagnetic waves in the visible or near-infrared part of the spectrum can be realized in optomagnonic cavities
- The typical configuration for implementing magneto-optical coupling is through spherical cavities composed of bismuth-substituted yttrium iron garnets (Bi:YIG), which exhibit gyroelectric properties in the near-infrared
- Spheroidal cavity shape presents significant interest, due to experimental considerations and the occurrence of shape defects during the manufacturing of spherical cavities

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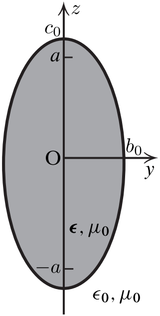
Configuration under study

Prolate spheroid composed of magneto-optical material



c_0: semi-major axis
b_0: semi-minor axis
α: semi-focal distance
$h = \alpha / c_0$: eccentricity

- Gyroelectric permitivitty tensor due to external magnetic bias

$$\boldsymbol{\epsilon} = \epsilon_0 \begin{bmatrix} \epsilon & ig & 0 \\ -ig & \epsilon & 0 \\ 0 & 0 & \epsilon \end{bmatrix}$$

- Scattering formulation with impinging plane EM wave $\mathbf{E}^{\mathrm{inc}} = \mathbf{y} e^{ik_0(x \sin\theta_0 + z\cos\theta_0)}$

Field expansions

Incident field: $$\mathbf{E}^{\mathrm{inc}}(\mathbf{r_s}) = \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} \left[C_{mn}(c,\theta_0)\mathbf{M}_{mn}^{r(1)}(c,\mathbf{r_s}) + D_{mn}(c,\theta_0)\mathbf{N}_{mn}^{r(1)}(c,\mathbf{r_s}) \right],$$

Scattered field: $$\mathbf{E}^{\mathrm{sc}}(\mathbf{r_s}) = \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} \left[A_{mn}\mathbf{M}_{mn}^{r(3)}(c,\mathbf{r_s}) + B_{mn}\mathbf{N}_{mn}^{r(3)}(c,\mathbf{r_s}) \right],$$

where $\mathbf{r_s} = (\xi, \eta, \varphi)$ the spheroidal coordinates, $c = k_0\alpha$, $\mathbf{M}_{mn}^{r(j)}$ and $\mathbf{N}_{mn}^{r(j)}$ the complex spheroidal eigenvectors of the first ($j = 1$) and third kind ($j = 3$).

2 Solution of the problem

Field expansions

- Employ spherical eigenvector expansion of the electric field in the gyroelectric region [Li and Ong, IEEE TAP, 2011]

Internal field:
$$\mathbf{E}^{\rm int}(\mathbf{r}) = \sum_{m=-\infty}^{\infty} \sum_{\substack{n=|m| \\ (m,n)\neq(0,0)}}^{\infty} \overline{E}_{mn} \sum_{l=1}^{\infty} a_l \Big[c_{mnl}\mathbf{m}_{mn}^{(1)}(k_l,\mathbf{r}) + d_{mnl}\mathbf{n}_{mn}^{(1)}(k_l,\mathbf{r}) + \frac{\overline{w}_{mnl}}{\lambda_l}\mathbf{l}_{mn}^{(1)}(k_l,\mathbf{r}) \Big] + \sum_{l=1}^{\infty} a_l \frac{w_{00l}}{\lambda_l}\mathbf{l}_{00}^{(1)}(k_l,\mathbf{r}),$$

where $\mathbf{r} = (r, \theta, \varphi)$ the spherical coordinates, $\overline{E}_{mn}$ known normalization constant, and $\mathbf{m}_{mn}^{(1)}$, $\mathbf{n}_{mn}^{(1)}$, $\mathbf{l}_{mn}^{(1)}$ the complex spherical eigenvectors of the first kind, $k_l = k_0\sqrt{\epsilon/\lambda_l}$, whereas c_{mnl}, d_{mnl}, $\overline{w}_{mnl}$, w_{00l}, λ_l are known quantities, obtained by solving an eigenvalue problem, the coefficient matrix of which depends on the permittivity tensor elements.

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2 Solution of the problem

Field expansions

- Transform spherical expansion of $\mathbf{E}^{\rm int}$ into one in terms of spheroidal eigenvectors [Cooray and Ciric, COMPEL, 1989]

Internal field: $$\mathbf{E}^{\rm int}(\mathbf{r_s}) = \sum_{m=-\infty}^{\infty}\sum_{n=|m|}^{\infty}\sum_{l=1}^{\infty} a_l \Big[C_{mnl}\mathbf{M}_{mn}^{r(1)}(c_l,\mathbf{r_s}) + D_{mnl}\mathbf{N}_{mn}^{r(1)}(c_l,\mathbf{r_s}) + \frac{\overline{W}_{mnl}}{\lambda_l}\mathbf{L}_{mn}^{(1)}(c_l,\mathbf{r_s})\Big] + \sum_{l=1}^{\infty}{\sum_{\ell=0}^{\infty}}' a_l \frac{w_{00l}}{\lambda_l}\Gamma_{00\ell}(c_l)\mathbf{L}_{0\ell}^{(1)}(c_l,\mathbf{r_s}),$$

where $\mathbf{L}_{mn}^{(1)}$ the irrotational complex spheroidal eigenvector of the first kind, $c_l = k_l\alpha$, whereas C_{mnl}, D_{mnl}, $\overline{W}_{mnl}$, $\Gamma_{00\ell}$ are known quantities.

- Respective magnetic fields $\mathbf{H}^{\rm inc}$, $\mathbf{H}^{\rm sc}$ and $\mathbf{H}^{\rm int}$ obtained by Faraday's law $\mathbf{H} = -i/(\omega\mu_0)\nabla\times\mathbf{E}$

2 Solution of the problem

Boundary conditions at spheroid's surface

$$\hat{n} \times \left[\mathbf{E}^{\mathrm{sc}}(\mathbf{r_s}) + \mathbf{E}^{\mathrm{inc}}(\mathbf{r_s}) - \mathbf{E}^{\mathrm{int}}(\mathbf{r_s})\right]_{\mathbf{r_s}\in S} = 0,$$

$$\hat{n} \times \left[\mathbf{H}^{\mathrm{sc}}(\mathbf{r_s}) + \mathbf{H}^{\mathrm{inc}}(\mathbf{r_s}) - \mathbf{H}^{\mathrm{int}}(\mathbf{r_s})\right]_{\mathbf{r_s}\in S} = 0.$$

- Four sets of linear equations involving the unknown field expansion coefficients $\{A_{mn}, B_{mn}, a_l\} \longrightarrow$ linear system of the form $\mathbb{A}(x_0)\mathbf{v} = \mathbf{b}$
- $\mathbf{v} = [A_{mn}, B_{mn}, a_l]^T$ is the vector of unknown expansion coefficients, $\mathbf{b}$ is the excitation vector whose components depend on the expansion coefficients of the incident wave $C_{mn}(c, \theta_0)$, $D_{mn}(c, \theta_0)$, and $\mathbb{A}(x_0)$ is the system matrix
- $x_0 = k_0 c_0$: **normalized wavenumber**

2 Solution of the problem

Resonance problem

- Set $\mathbf{b} = 0$, i.e., consider zero excitation, to investigate the resonance problem $\longrightarrow \mathbb{A}(x_0)\mathbf{v} = 0$
- In order for the system to have non-trivial solutions $\longrightarrow \det \mathbb{A}(x_0) = 0$
- Employ an efficient root-finding algorithm [Zouros, Comput. Phys. Comm., 2018] and find complex resonant wavenumbers x_0
- Respective complex eigenfrequencies $f = x_0/(2\pi c_0\sqrt{\epsilon_0\mu_0})$

3 Numerical Results

Eigenfrequency calculation

Spherical cavity—comparison with shape perturbation technique of [Kolezas *et al*, IEEE JSTQE, 2019]

Table: Normalized eigenfrequencies x_0 for magneto-optic spherical cavity with $h = 0$. Values of parameters: $\epsilon = 5.5$ and $g = 0.02$.

Mode index	x_0 [this work]	$\mathrm{Re}\{x_0\}$ [Kolezas *et al*, IEEE JSTQE, 2019]
$m = 2$	$6.34087 - 6.96427 \times 10^{-5} i$	6.34087
$m = 1$	$6.34097 - 6.96901 \times 10^{-5} i$	6.34097
$m = 0$	$6.34105 - 6.97215 \times 10^{-5} i$	6.34106
$m = -1$	$6.34114 - 6.97310 \times 10^{-5} i$	6.34113
$m = -2$	$6.34121 - 6.97150 \times 10^{-5} i$	6.34121

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Eigenfrequency calculation

Slightly perturbed spherical cavity—comparison with shape perturbation technique of [Kolezas *et al*, IEEE JSTQE, 2019]

Table: Normalized eigenfrequencies x_0 for magneto-optic spheroidal cavity with $h = 0.01$. Values of parameters: $\epsilon = 5.5$ and $g = 0.02$.

Mode index	x_0 [this work]	$\mathrm{Re}\{x_0\}$ [Kolezas *et al*, IEEE JSTQE, 2019]
$m = 2$	$6.34104 - 6.96459 \times 10^{-5} i$	6.34103
$m = 1$	$6.34113 - 6.96928 \times 10^{-5} i$	6.34112
$m = 0$	$6.34121 - 6.97247 \times 10^{-5} i$	6.34119
$m = -1$	$6.34130 - 6.97304 \times 10^{-5} i$	6.34127
$m = -2$	$6.34137 - 6.97147 \times 10^{-5} i$	6.34139

Eigenfrequency calculation

Table: Normalized eigenfrequencies x_0 for magneto-optic spheroidal cavity with $h = 0.1$. Values of parameters: $\epsilon = 5.5$ and $g = 0.02$.

Mode index	x_0—[this work]
$m = 2$	$6.35755 - 7.05128 \times 10^{-5} i$
$m = 1$	$6.35726 - 7.04829 \times 10^{-5} i$
$m = 0$	$6.35719 - 7.04199 \times 10^{-5} i$
$m = -1$	$6.35736 - 7.03187 \times 10^{-5} i$
$m = -2$	$6.35776 - 7.01947 \times 10^{-5} i$

3 Numerical Results

Eigenfrequency calculation

Table: Normalized eigenfrequencies x_0 for magneto-optic spheroidal cavity with $h = 0.2$. Values of parameters: $\epsilon = 5.5$ and $g = 0.02$.

Mode index	x_0—[this work]
$m = 2$	$6.41003 - 8.00140 \times 10^{-5} i$
$m = 1$	$6.40851 - 8.00846 \times 10^{-5} i$
$m = 0$	$6.40796 - 7.98339 \times 10^{-5} i$
$m = -1$	$6.40840 - 7.92740 \times 10^{-5} i$
$m = -2$	$6.40981 - 7.84405 \times 10^{-5} i$

4 Conclusions

- Method based on spheroidal eigenvector formulation for the calculation of the eigenfrequencies of prolate spheroidal magneto-optic cavities is proposed
- Our approach allows for the calculation of the eigenfrequencies with increased accuracy but turns out to be time consuming when large changes in the eccentricity h are considered
- Small changes in h lead to relatively large shifts in the eigenfrequencies
- Straightforward extension to oblate spheroidal cavities

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Thank you for your attention!