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Elliptically inhomogeneous plane wave impinging on an infinite number of parallel cylinders

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Introduction

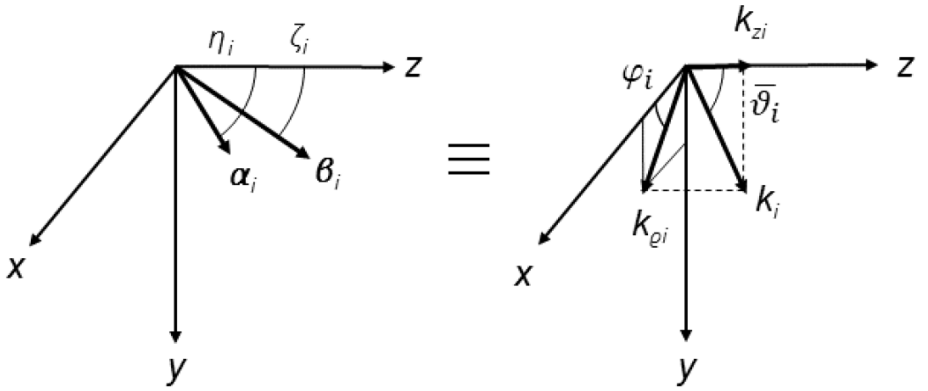
The work is motivated by the large number of applications in the biological and chemical fields

The scattered electromagnetic field by an indefinite number of infinite circular cylinders is analyzed through an application of the generalized Vector Cylinder Harmonics (VCH) expansion. The scenario described above is represented by an exact mathematical model that considers the so-called complex-angle formalism reaching a superposition of VCH and the Foldy-Lax Multiple Scattering Equations (FLMSE) to take into account the multi-scattering process between the cylinders. The validation of the method was performed with a comparison between the numerical results based on the Finite Element Method (FEM) and a homemade Matlab code.

Approach to the model

From literature, two formalisms are known to be used for representing an inhomogeneous wave propagating in a lossy medium.

The former and also the one with best characteristics is the formalism known as Adler-Chu-Fano formulation; its propagation vector has a complex nature with $k_i = \beta_i + i\alpha_i$ represented by the phase and attenuation vectors, β_i and α_i, respectively. The latter, once again, has a complex propagation vector represented by the superposition of real and imaginary parts $k_i = k_R + ik_I$, which forms a complex angle with an axis of the Cartesian reference system



The left figure represents the complex wave vector of an inhomogeneous plane wave with the phase and attenuation vectors. The right figure represents the same vectors with the complex-angle formulation.

Approach to the model

This study demonstrates that using a superposition of basic cylindrical waves to represent the field through the use of the complex-angle formalism can be expressed with relative simplicity. The following wave, in which the vectors α_i and β_i are forming the angles ζ_i and η_i with the *z*-axis is also placed on the same plane passing through the *z*-axis, and they are creating a real angle φ with the *x.*

$$\cos\vartheta_R = \frac{k_{R_{N+1}}\beta_{N+1}\cos\xi_{N+1} + k_{I_{N+1}}\alpha_{N+1}\cos\eta_{N+1}}{\sqrt{k^2_{R_{N+1}}\beta^2_{N+1} - k^2_{I_{N+1}}\alpha^2_{N+1} + 2(k_{R_{N+1}}k_{I_{N+1}})^2}} \qquad tg\,2\vartheta_I = \frac{2\beta_{N+1}\alpha_{N+1}}{k^2_{N+1}}$$

where η and ζ are the angles that the vectors α and β, respectively, form with the *z*-axis

Approach to the model

Any obliquely polarized elliptical field, with respect to the surface of a cylinder, can be represented as a linear combination of two components, one vertical and one horizontal, each multiplied by its polarization coefficient (E_{vi} and E_{hi}, respectively):

$$\mathbf{E}(\mathbf{r}) = \left[E_{vi}\mathbf{v}_0(\bar{\vartheta}_i, \varphi_i) + E_{hi}\mathbf{h}_0(\bar{\vartheta}_i, \varphi_i)\right] e^{i\mathbf{k}\cdot\mathbf{r}} =$$

$$= \sum_{m=-\infty}^{+\infty} \left[a_m \mathbf{M}_m(k^*\mathbf{r}) + b_m \mathbf{N}_m(k^*\mathbf{r})\right]$$

with

$$a_m = \frac{E_{hi}}{k_\rho}(-i)^{m-1} e^{-im\varphi_i}$$

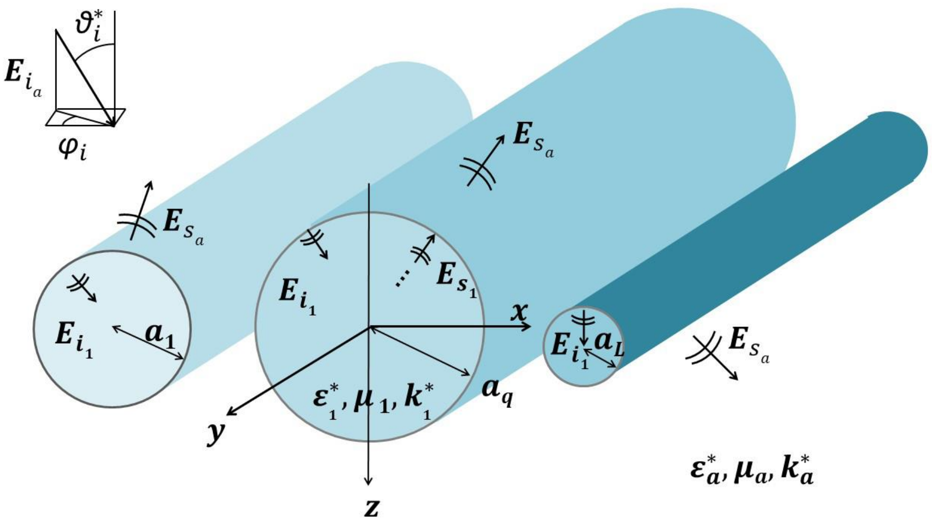
$$b_m = -\frac{E_{vi}}{k_\rho}(-i)^{m} e^{-im\varphi_i}$$

$$\mathbf{k}_i = k^* \left(\sin\bar{\vartheta}_i \cos\varphi_i \mathbf{x}_0 + \sin\bar{\vartheta}_i \varphi_i \mathbf{y}_0 + \cos\bar{\vartheta}_i \mathbf{z}_0\right)$$

$$\mathbf{M}_m = (im\frac{Z_m(k_\rho \rho)}{\rho}\rho_0 - k_\rho \frac{\partial Z_m(k_\rho \rho)}{\partial \rho}\varphi_0) e^{im\varphi} e^{ik_z z - i\omega}$$

$$\mathbf{N}_m = (i\frac{k_z k_\rho}{k}\frac{\partial Z_m(k_\rho \rho)}{\partial \rho}\rho_0 - \frac{mk_z}{k}\frac{Z_m(k_\rho \rho)}{\rho}\varphi_0 +$$

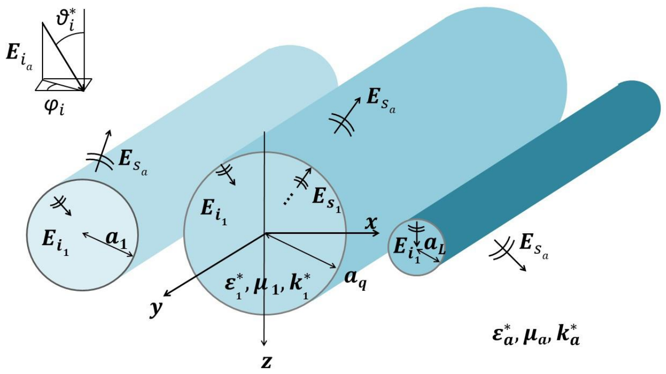
$$+ \frac{k_\rho^2}{k} Z_m(k_\rho \rho)\mathbf{z}_0) e^{im\varphi} e^{ik_z z - i\omega t}$$



Approach to the model

An arbitrary number L of dielectric cylinders, with relative permittivities ε_j, with $j = 1,\ldots,N$, infinite length, and radii r_j in a free-space filled by a lossy medium, in general dissipative, with relative permittivity ε_e, relative permeability μ_e, and electric conductivity σ_e are considered. The incident field, as usual, is an elliptically polarized inhomogeneous plane wave. In order to apply the Foldy-Lax Multiple scattering equations, the external field on the surface of the q-th cylinder, also called the exciting field, needs to be taken into consideration. The exiting field is the superposition of the incident field and all the scattered fields by the cylinders:

$$\mathbf{E}_{ex}^{q} = \mathbf{E}_{i} + \sum_{\substack{p=1 \\ p\neq q}}^{L} \mathbf{E}_{s}^{p}.$$



Approach to the model

The incident field can be expressed as a function of vector cylindrical harmonics centered on the *q*-th cylinder

$$\mathbf{E}_i(k\rho_q) = [E_{v0}v + E_{h0}h]\, e^{i\mathbf{k}_0\cdot\rho_q} e^{i\mathbf{k}_0\cdot\rho_{0q}} = \sum_{m=-\infty}^{+\infty} \left[a_m \mathbf{M}_m^{(1)}(\rho - \rho_q) + b_m \mathbf{N}_m^{(1)}(\rho_{0q})\right] e^{i\mathbf{k}_0\cdot\rho_q}$$

The exiting field of the *q*-th cylinder is:

$$\mathbf{E}_{ex}^q(k\rho_q) = \sum_{m=-\infty}^{+\infty} \left[w_m^q \mathbf{M}_m^{(1)}(\rho_{0q}) + v_m^q \mathbf{N}_m^{(1)}(\rho_{0q})\right]$$

while the scattered electric field from $p \neq q$-th cylinder is:

$$\mathbf{E}_s^p(\rho_p) = \sum_{m'=-\infty}^{+\infty} \left[T_{m'}^M w_{m'}^p \mathbf{M}_m^{(3)}(\rho_{0p}) + T_{m'}^N v_{m'}^p \mathbf{N}_m^{(3)}(\rho_{0p})\right]$$

having indicated with *T* the scattering coefficients in dielectric cylinder case, i.e. the T-matrix coefficients

Approach to the model

Applying the Addition theorem on the VCHs function, we obtain:

$$M_{m'}^{(3)}(\rho_{0q}) = \sum_{m} H_{m-m'}^{(1)}(k\rho_{qp}) e^{-i(m-m')\varphi_{pq}} M_{m}^{(1)}(\rho_{0q})$$

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$$M_{m'}^{(1)}(\rho_{0q}) = \sum_{m} J_{m-m'}^{(1)}(k\rho_{qp}) e^{-i(m-m')\varphi_{pq}} M_{m}^{(1)}(\rho_{0q})$$

$$N_{m'}^{(1)}(\rho_{0q}) = \sum_{m} J_{m-m'}^{(1)}(k\rho_{qp}) e^{-i(m-m')\varphi_{pq}} N_{m}^{(1)}(\rho_{0q})$$

Replacing all fields inside the FLMSEs and using the orthogonal properties of the VCHs, the following linear system is obtained:

$$w_m^q = \tilde{a}_m + \sum_{m'=-\infty}^{+\infty} \sum_{\substack{p=1 \\ p\neq q}} A_{mm'} T_{m'}^{M} w_{m'}^{p}$$

$$v_m^q = \tilde{b}_m + \sum_{m'=-\infty}^{+\infty} \sum_{\substack{p=1 \\ p\neq q}} A_{mm'} T_{m'}^{N} v_{m'}^{p}.$$

Approach to the model

At this point, the linear system can be solved and the coefficients w_m^q and v_m^q determined. Being the scattered field by the q-th cylinder writable as a superposition of VCHs, as:

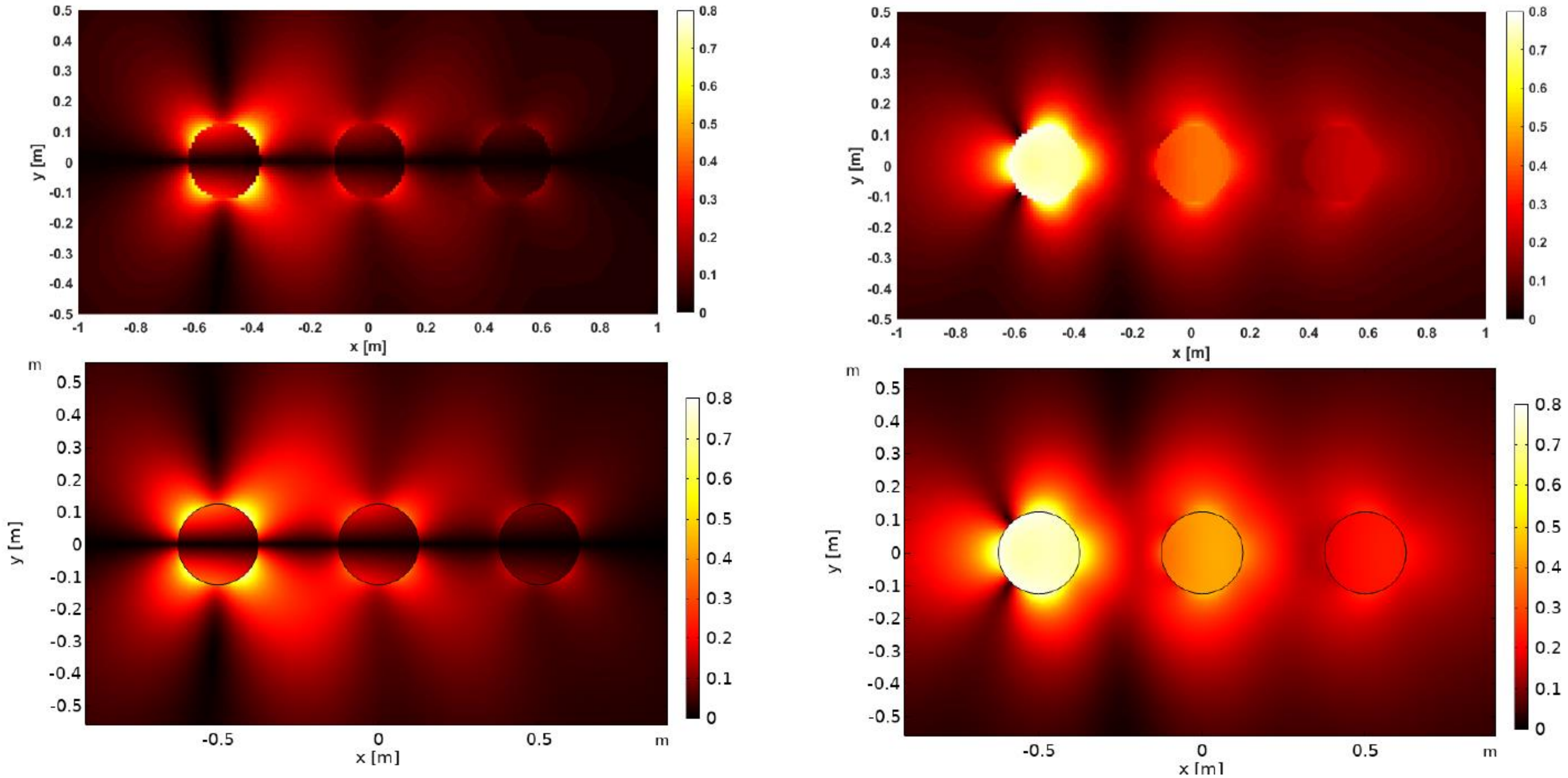
$$\mathbf{E}_s^q = \sum_{m=-\infty}^{+\infty} \left[e_m^q \mathbf{M}_m^{(3)}(k, \rho_{0q}) + f_m^q \mathbf{N}_m^{(3)}(k, \rho_{0q}) \right]$$

the coefficients of the q-th cylinder can be written as follows:

$$e_m^q = T_m^M w_m^q$$
$$f_m^q = T_m^N v_m^q$$

Results

A comparison as a result of the validation process was performed both on the determined formulation and on a canonical case of electromagnetic scattering.



The results obtained with Matlab (top) and Comsol (bottom) in the case of k_e = 1-i [1/m], for the environment and k_c = 2-0.5i [1/m] for all cylinders. Rigth: E_x component, left E_y

Conclusions

An accurate method to obtain an expansion of an inhomogeneous elliptically polarized plane wave in terms of vectorial cylinder harmonics to solve the multiscattering by an ensemble of cylinders is presented. The determination of the expansion coefficients and the application of the so-called Foldy-Lay equations for the use of the complex-angle formalism contribute to the determination of the solution to the electromagnetic problem. A light and elegant formalism was achieved with this approach. The procedure was validated with some numerical results as well as with comparisons through simulations in the COMSOL environment.

END

Thanks for the attention