



POLARIMETRIC TWO-SCALE MODEL FOR ROUGH SURFACE BISTATIC SCATTERING EVALUATION

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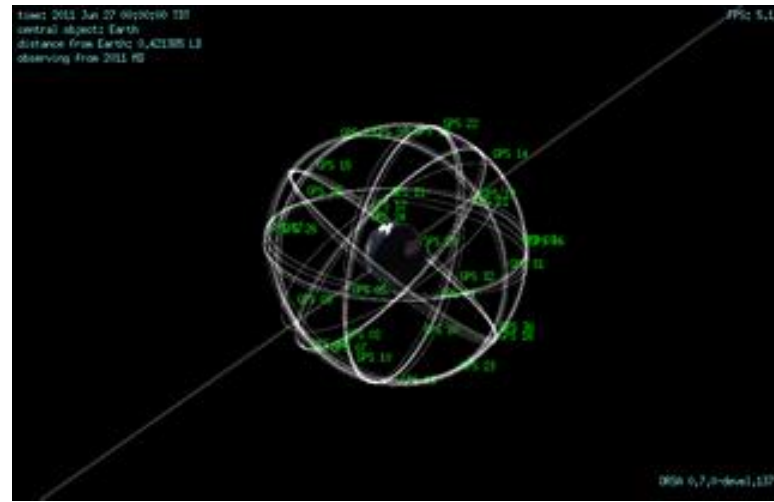
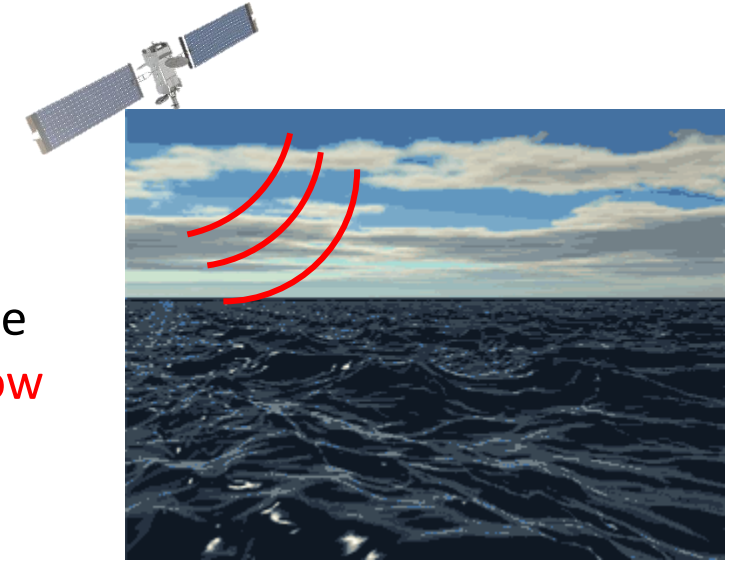


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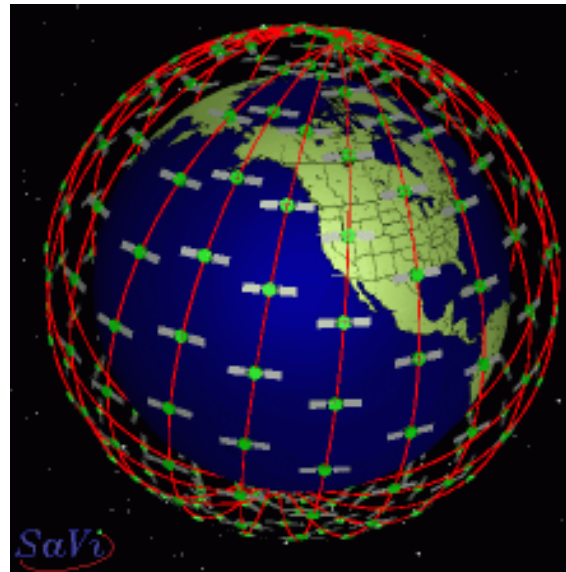
Introduction and motivation

- **Microwave** sea observations are of fundamental importance, since they allow retrieving parameters of the **sea** and of **objects on the sea** surface.



- In recent years the interest in GNSS reflectometry (**GNSS-R**) is increasing, due to the promise of **low revisit times** and **low costs**.

- For similar reasons, **low-orbit small-satellite constellations** have become a hot research topic, too



- Appropriate **electromagnetic models** are required to **design** the system, **assess** its performance through **simulation tools**, and to support the development of adequate **inversion techniques**.

Introduction and motivation

What surface model?

Sea surface



**Anisotropic
roughness**

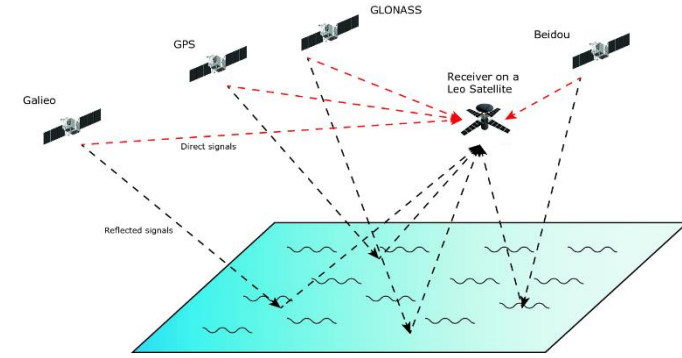


What scattering model?

GNSS or
constellation



**Bistatic
scattering**



What applications?

Sea state



**Specular or near specular
acquisition geometry**

**Wide
range of
scattering
angles**

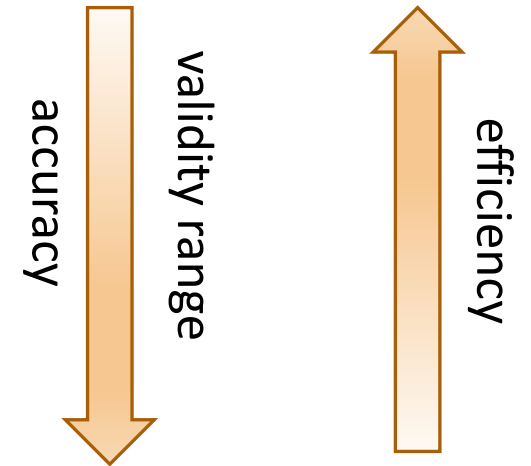
Object
detection



**Far from specular
acquisition geometry**

Models for scattering from natural (randomly rough) surfaces

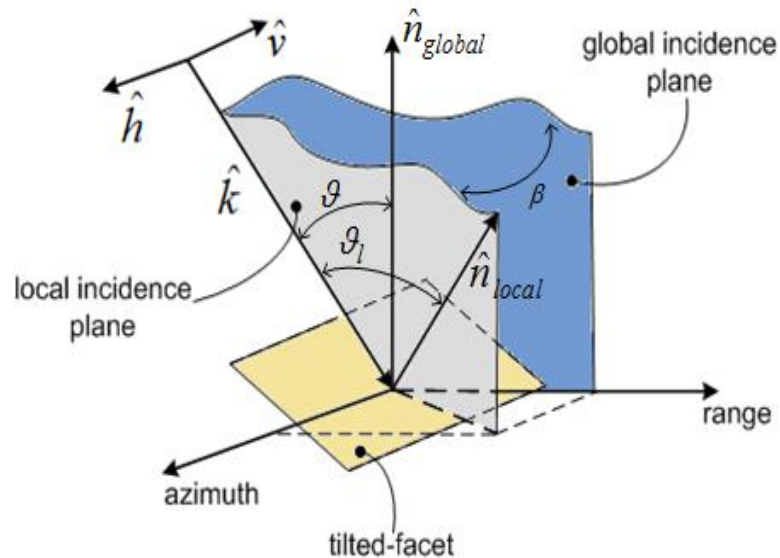
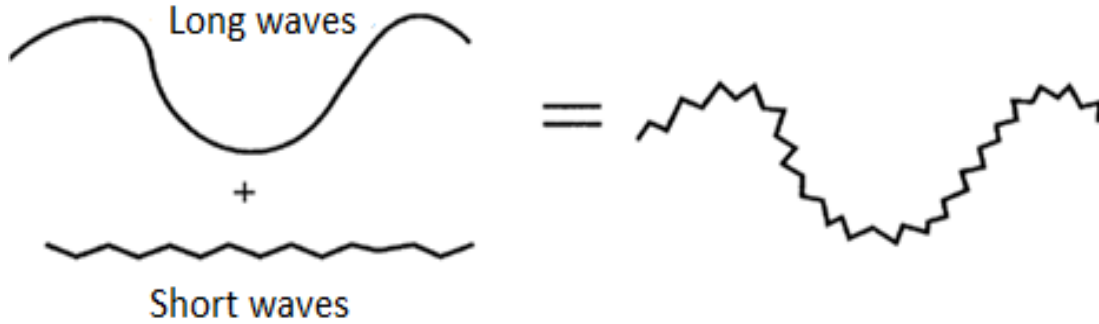
- Approximate analytical, closed-form (SPM, GO, empirical)
- Approximate analytical/numerical (TSM, SPM2, SSA, SSA2)
- “Exact” fully numerical (MoM + Monte Carlo simulation)



TSM is **widely used** to model the scattering from the **sea surface**

Introduction and motivation

Two-Scale Model (TSM)



Total NRCS = **large scale** roughness NRCS (computed via GO) + **small scale** roughness NRCS (computed via SPM)

GO dominates at low incidence angles

SPM dominates at intermediate/high incidence angles

Range of validity: union of GO and SPM ones.

no cross-pol and de-pol unless SPM term is **averaged** over **random slopes** of tilted mean plane

Average over slopes -> **numerical integration!**

J. W. Wright, "A New Model for Sea Clutter", *IEEE Trans. Antennas Propagat.*, vol. 16, pp. 217-223, 1968.

G. R. Valenzuela, "Scattering of Electromagnetic Waves from a Tilted Slightly Rough Surface", *Radio Sci.*, vol. 3, pp. 1057-1066, 1968.

PTSM

- Almost ten years ago the **Polarimetric Two-Scale Model (PTSM)** was introduced¹, allowing for **closed-form** evaluation of the average integral, via a moderate slope approximation.
- PTSM allows accounting for **cross-** and **de-polarisation** effects actually present in measured data even when **surface scattering** is the **only** present mechanism.
- PTSM has been used to devise **soil moisture** retrieval schemes for **bare soils**¹.
- Recently PTSM has been extended to the case of the **anisotropic sea surface (A-PTSM)**², but in backscattering configuration only.

Extension to the case of bistatic scattering configuration is considered in this work.

¹A. Iodice, A. Natale, D. Riccio, “Retrieval of Soil Surface Parameters via a Polarimetric Two-Scale Model”, *IEEE Trans. Geosci. Remote Sens.* vol. 49, no. 7, pp. 2531-2547, July 2011.

²G. Di Martino, A. Iodice, D. Riccio, “Closed-Form Anisotropic Polarimetric Two-Scale Model for Fast Evaluation of Sea Surface Backscattering”, *IEEE Trans. Geosci. Remote Sens.*, vol. 57, no. 8, pp. 6182-6194, Aug. 2019.

Theory

Surface description

Small-scale roughness:

High-frequency part of the directional **Elfouhaily spectrum**

$$W_{2D}(\kappa, \varphi) = W(\kappa)\Phi(\kappa, \varphi)$$

$$W(\kappa) = \frac{\pi\alpha_m c_m}{c \kappa^4} \exp\left[-\frac{1}{4}\left(\frac{\kappa}{\kappa_m} - 1\right)^2\right]$$

$$\Phi(\kappa, \varphi) = 1 + \Delta(\kappa) \cos[2(\varphi_w - \varphi)]$$

$$c = \sqrt{\frac{g}{\kappa} \left[1 + \left(\frac{\kappa}{\kappa_m}\right)^2\right]} \quad u^* = \sqrt{C_d} u_{10}$$

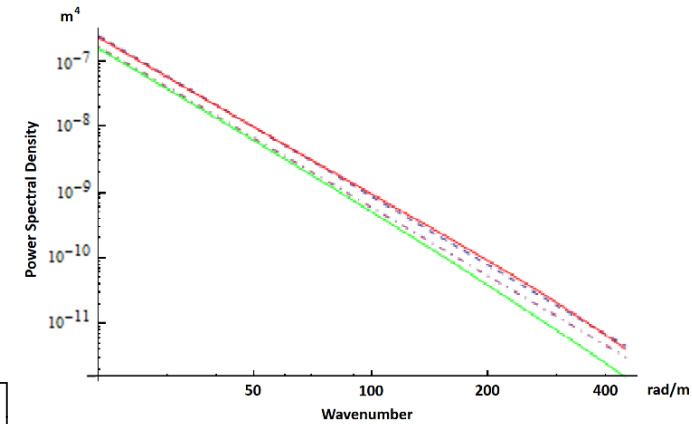
$$\alpha_m = \begin{cases} 0.01[1 + \ln(u^*/c_m)] & \text{for } u^* \leq c_m \\ 0.01[1 + 3\ln(u^*/c_m)] & \text{for } u^* > c_m \end{cases}$$

$$\kappa_m = 363 \text{ m}^{-1} \quad \Delta(\kappa) = \tanh\left[0.173 + 4(c/c_p)^{2.5} + a_m(c_m/c)^{2.5}\right]$$

$$c_m = 0.23 \text{ m/s} \quad c_p \cong u_{10}/0.84 \text{ and } a_m = 0.13u^*/c_m$$

if $20 \text{ m}^{-1} < \kappa < 200 \text{ m}^{-1}$

$$c \cong \sqrt{g/\kappa} \quad W(\kappa) \cong \frac{S_0}{\kappa^{3.5}} \quad \Phi(\kappa, \varphi) \cong \Phi(\varphi) = 1 + \Delta(\kappa_0) \cos[2(\varphi_w - \varphi)] \quad \Delta(\kappa) < \sim 0.2$$



u_{10} : wind velocity
 φ_w : wind direction

Surface description

Large-scale roughness:

Up-wind and cross-wind slopes s_{up} and s_{cross} : **independent zero-mean Gaussian variables** with σ_{up} and σ_{cross} standard deviations.

Katzberg model for $f=1.5$ GHz (GNSS)

$$\sigma_{up0}^2 = 0.45 \left[0.00316 \cdot 6 \ln(u_{10}) \right]$$

$$\sigma_{cross0}^2 = 0.45 \left[0.003 + 0.00192 \cdot 6 \ln(u_{10}) \right]$$

Evaluation for generic f :

$$\begin{aligned} \sigma_{up,cross}^2 &\cong \sigma_{up0,cross0}^2 + \frac{1}{4\pi^2} \int_0^{2\pi} \int_{\kappa_{cut0}}^{\kappa_{cut}} \kappa^2 \cos^2(\varphi - \varphi_w - \psi_{up,cross}) W(\kappa) \Phi(\varphi) \kappa d\kappa d\varphi = \\ &= \sigma_{up0,cross0}^2 + \frac{S_0}{2\pi} \left(1 \pm \frac{\Delta(\kappa_0)}{2} \right) \left(\sqrt{\kappa_{cut}} - \sqrt{\kappa_{cut0}} \right) \quad \psi_{up} = 0, \psi_{cross} = \pi/2 \end{aligned}$$

Azimuth and range slopes s_a and s_r : **correlated zero-mean Gaussian variables** with σ_a and σ_r standard deviations and ρ correlation coefficient. $s_a, s_r \sim N\left(0; \sigma_a^2, \sigma_r^2, \rho\right)$

$$\sigma_r^2 = \sigma_{up}^2 \cos^2 \varphi_w + \sigma_{cross}^2 \sin^2 \varphi_w$$

$$\sigma_a^2 = \sigma_{cross}^2 \cos^2 \varphi_w + \sigma_{up}^2 \sin^2 \varphi_w$$

$$\rho = \frac{1}{2} \sin 2\varphi_w \frac{\sigma_{cross}^2 - \sigma_{up}^2}{\sigma_r \sigma_a}$$

Bistatic A-PTSM

1) Compute **tilted surface's** polarimetric covariance matrix via **SPM** in terms of the **local** incidence ϑ_{li} and scattering $\vartheta_{ls}, \varphi_{ls}$ angles, and of **rotation angles** β_i and β_s of incidence and scattering planes

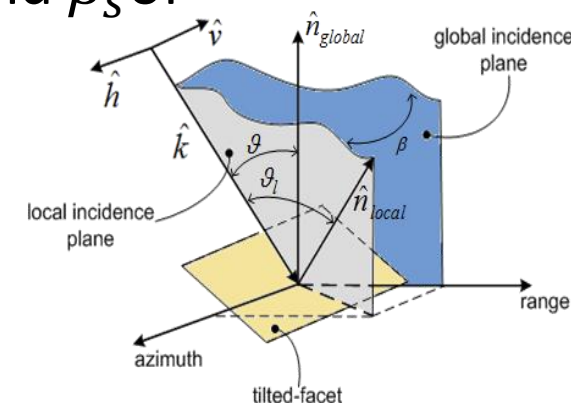
2) Express $\vartheta_{li}, \vartheta_{ls}, \varphi_{ls}, \beta_i$ and β_s in terms of global incidence ϑ_i and scattering ϑ_s, φ_s angles and of local surface slopes s_x and s_y

3) **Second order expansion** of tilted surface's covariance matrix around $s_x = 0$ and $s_y = 0$

4) Averaging tilted surface's NRCS and other entries of the covariance matrix over s_x and s_y by

using: $\langle s_x \rangle = \langle s_y \rangle = 0,$ $\langle s_x^2 \rangle = \sigma_x^2,$ $\langle s_y^2 \rangle = \sigma_y^2,$ and $\langle s_x s_y \rangle = \rho \sigma_x \sigma_y$

Expressions for ϑ_{li} and β_i are already available, while those for $\vartheta_{ls}, \varphi_{ls}$ and β_s are an **original contribution** of this work



Covariance matrix elements

$$\langle R_{pq,rs}^{SPM}(\vartheta_i, \vartheta_s, \varphi_s; s_x, s_y) \rangle_{s_x, s_y} \cong R_{pq,rs}^{SPM}(\vartheta_i, \vartheta_s, \varphi_s; \mathbf{0}, \mathbf{0}) + D_{2,0}^{pq,rs} \sigma_x^2 + D_{0,2}^{pq,rs} \sigma_y^2 + D_{1,1}^{pq,rs} \rho \sigma_x \sigma_y$$

$$\kappa_y = -k \sin \vartheta_s \sin \varphi_s$$

$$\kappa_x = -k \sin \vartheta_s \cos \varphi_s + k \sin \vartheta_i$$

$$\bar{\kappa} = \sqrt{\kappa_x^2 + \kappa_y^2}$$

$$\bar{\varphi} = \arctan(\kappa_y / \kappa_x)$$

Bragg coefficients

$$\left\{ \begin{array}{l} F_{hh} = \frac{(\varepsilon_r - 1) \cos \varphi_s}{(\cos \varphi_s + \sqrt{\varepsilon_r - \sin^2 \vartheta_s}) (\cos \vartheta_i + \sqrt{\varepsilon_r - \sin^2 \vartheta_i})} \\ F_{hv} = \frac{\sin \varphi_s [(\varepsilon_r - 1) (\sqrt{\varepsilon_r - \sin^2 \vartheta_s})]}{(\sqrt{\varepsilon_r - \sin^2 \vartheta_s} + \varepsilon_r \cos \vartheta_s) (\cos \vartheta_i + \sqrt{\varepsilon_r - \sin^2 \vartheta_i})} \\ F_{vh} = \frac{\sin \varphi_s [(\varepsilon_r - 1) (\sqrt{\varepsilon_r - \sin^2 \vartheta_i})]}{(\sqrt{\varepsilon_r - \sin^2 \vartheta_i} + \varepsilon_r \cos \vartheta_i) (\cos \vartheta_s + \sqrt{\varepsilon_r - \sin^2 \vartheta_s})} \\ F_{vv} = \frac{(\varepsilon_r - 1) [\sqrt{\varepsilon_r - \sin^2 \vartheta_i} \sqrt{\varepsilon_r - \sin^2 \vartheta_s} \cos \varphi_s - \varepsilon_r \sin \vartheta_i \sin \vartheta_s]}{(\varepsilon_r \cos \vartheta_s + \sqrt{\varepsilon_r - \sin^2 \vartheta_s}) (\varepsilon_r \cos \vartheta_i + \sqrt{\varepsilon_r - \sin^2 \vartheta_i})} \end{array} \right.$$

Expansion coefficients

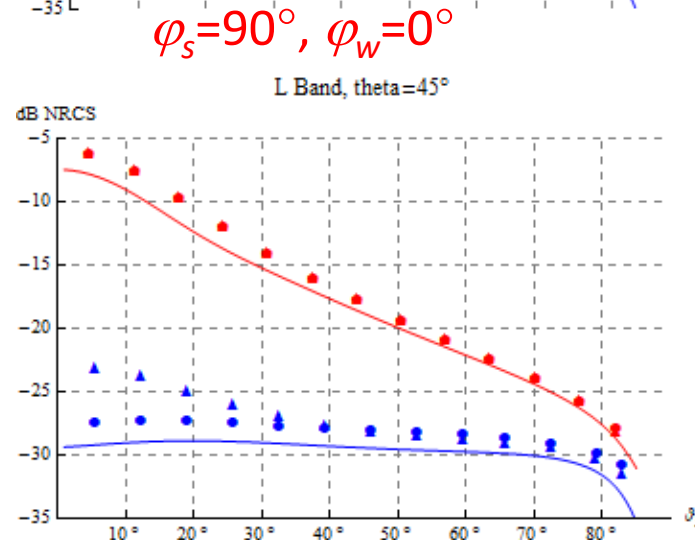
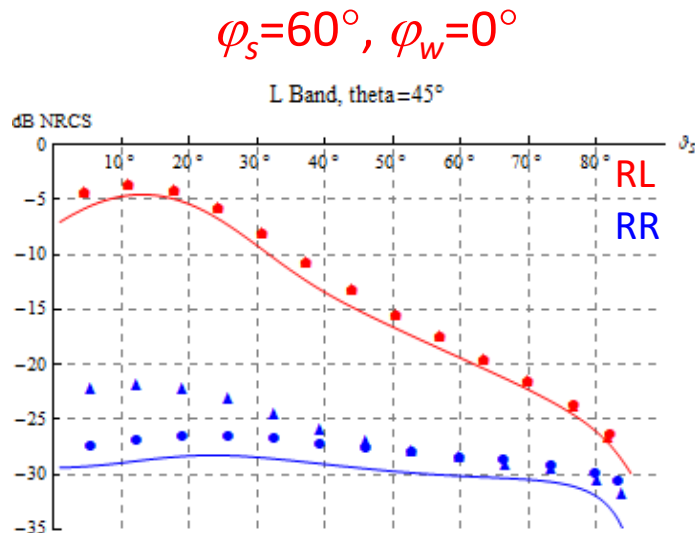
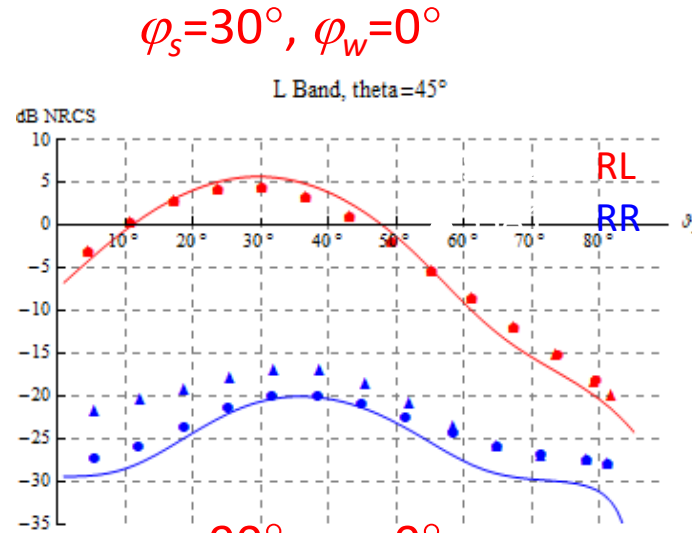
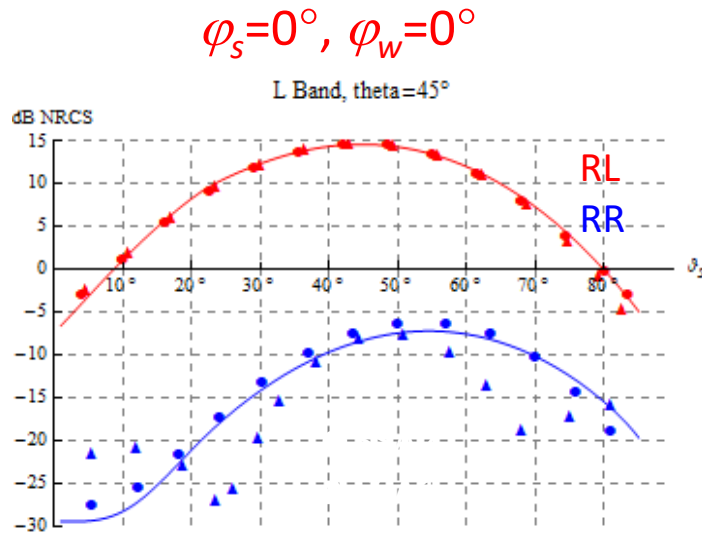
$$D_{k,n-k}^{pq,rs} = \frac{1}{n!} \binom{n}{k} \frac{\partial^n R_{pq,rs}^{SPM}}{\partial s_x^k \partial s_y^{n-k}} \Big|_{s_x=s_y=0}$$

Standard SPM elements covariance matrix

$$\begin{aligned} & R_{pq,rs}^{SPM}(\vartheta_i, \vartheta_s, \varphi_s, \mathbf{0}, \mathbf{0}) \\ &= \frac{4}{\pi} k^4 \cos^2 \vartheta_i \cos^2 \vartheta_s F_{pq}(\vartheta_i, \vartheta_s, \varphi_s) F_{rs}^*(\vartheta_i, \vartheta_s, \varphi_s) W_{2D}(\bar{\kappa}, \bar{\varphi}) \end{aligned}$$

Results

All experiments at L band, $\vartheta_i = 45^\circ$, and $u_{10} = 10$ m/s.



- SSA2
- ▲ SSA1

Bistatic A-PTSM results are closer to the SSA2 than to the SSA1 ones

SSA2¹ considers **multiple scattering** (up to second order), but it requires computationally intensive numerical evaluation of **fourfold integrals**

¹A. G. Voronovich and V. U. Zavorotny, "Full-polarization modeling of monostatic and bistatic radar scattering from a rough sea surface", *IEEE Trans. Antennas Propagat.*, vol. 62, no. 3, pp. 1362–1371, March 2014.

Conclusions

- Closed-form PTSM extended to the **anisotropic sea surface** case (**A-PTSM**) and to the **bistatic scattering** configuration
- **All elements** of the linear polarization polarimetric covariance matrix analytically expressed in **closed form**
- Reasonable agreement with **SSA2**, which is more accurate but **computationally intensive**
- For applications in which **computational efficiency** is important, **use A-PTSM!** (for instance, wind speed and direction retrieval, or, more in general, surface parameter retrieval).
- Extendable to the case of **agricultural anisotropic soil** surfaces, upon appropriate modeling of the roughness

THANK YOU
FOR YOUR ATTENTION!



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