Notes on Radiofrequency and Plasma Coupling in Inductive Plasma Ion Sources

Marco Cavenago[1]

[1]*INFN-Laboratori Nazionali di Legnaro (LNL), v. le dell'Università 2, I-35020, Legnaro (PD) Italy*

1) Introduction: inductive plasma heating

2) The rf conductivity in bounded plasmas (and the skin depth)

3) Unmagnetized plasmas

4) Magnetized plasmas

5) Simulations: workflow and (for magnetized plasma) results

6) Conclusion

Abstract

The advantage of contactless heating of plasmas (ionized gases) with radiofrequency (MHz) or microwaves (GHz) electromagnetic radiation well motivates a quest for both a theoretical understanding and reasonably fast numerical simulation techniques, capable to reproduce main experimental behavior and to help design of plasma ion sources.

The inductively coupled plasma model is briefly reviewed, with emphasis of the concepts of plasma rf conductivity and collision frequency ν_c. The so-called stochastic collision frequency (a local equivalent for rapid effects on electron trajectory as wall collisions and skin depth traversal) is discussed in the accepted limits (in the vast literature on waves and plasma interaction), and a new formula encompassing them is here proposed; moreover an universal graph of ν_c/ω vs plasma pressure is given. Finally the case of magnetized plasma with a typical transport model is outlined, describing the workflow of a code for its solution; equilibrium gas density is also calculated, showing a large reduction for rf coil current exceeding a threshold; position of peaks of electron density and rf field and rf absorption in plasma are discussed, with result for the efficiency.

1) Introduction

Ionized gases, also known as plasmas, need a continuos influx of energy, also known as heating, to maintain ionization. Heating with microwave or radiofrequency (not dissimilar from microwave food cooking) may seems expensive as compared with a simple arc between electrodes: but in the latter case, electrode sputtering inquinates plasmas and limits the device service life) . So rf/µwaves are often preferred in plasmas for ion production

the plasma angular frequency

$$\omega_p = \sqrt{n_e e^2/(m_e \varepsilon_0)}$$

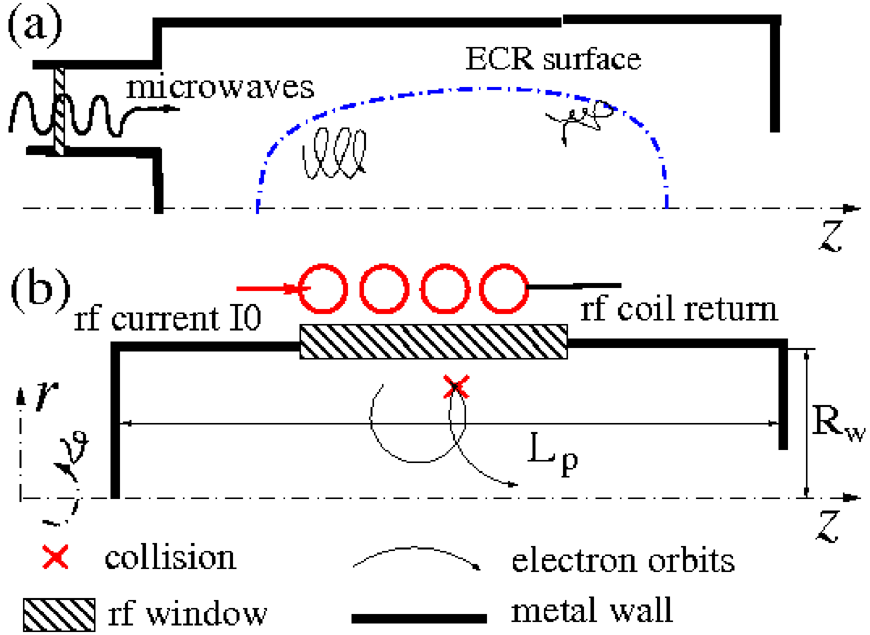
depends on electron density n_e

When n_e equals to the cutoff density n_c

$$n_c = m_e \varepsilon_0 \omega^2 / e^2$$

where ω is the angular microwave frequency, we have

$$\omega = \omega_p$$



(a) ECRIS (Electron Cyclotron Resonance Ion Source [4];
(b) Inductively Coupled Plasma (ICP).

Typically $n_e = 10^{18}$ m^{-3} in ion source center, so microwave source [ECRIS see Fig 1.(a)] are below cut off density and radiofrequency plasma (b) have density over the cutoff

1.2) RADIOFREQUENCY HEATING

Radiofrequency heating of a plasma involves the repeated acceleration and deceleration of free electrons inside plasma, in conditions hopefully tuned to increase the transfer of power from rf to electrons. HERE LET US ASSUME THAT RF POWER IS APPLIED WITH AN EXTERNAL COIL (roughly called ICP Inductively Coupled Plasma)

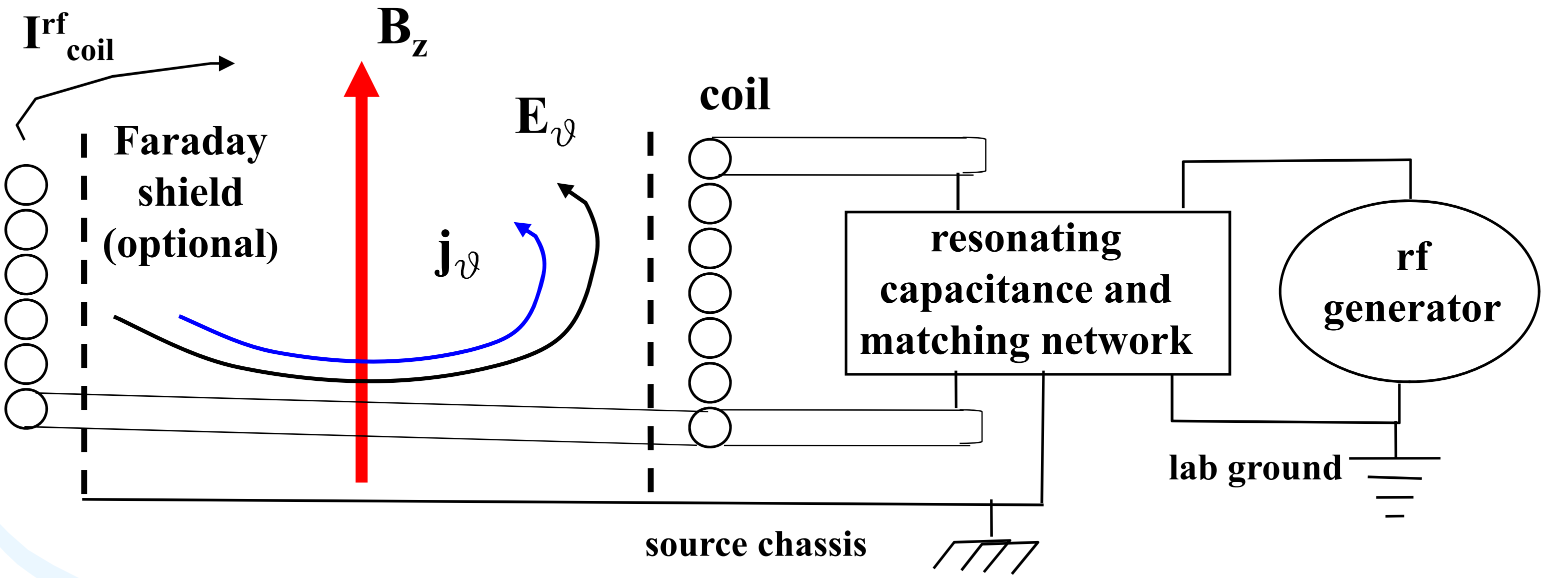


Figure: The simpler model: assuming that plasma behaves as the secondary of a transformer[2,8], the net current inside plasma balances the current in the coil; this model may give very large and somewhat unrealistic efficiencies

There are rf losses in the coil and the metal wall of the vacuum chamber, and in the Faraday shield when used.

1.3) Strategy for modeling and simulation of rf plasma; why not a simulation of everything (i.e. *ab initio Monte Carlo*)-

Collisions (events much shorter than one radiofrequency period) are important: without them, an electron will be periodically accelerated and decelerated by radiofrequency, with no energy gain in the time average

So we need to add collision effects to Lorentz force in motion equation.

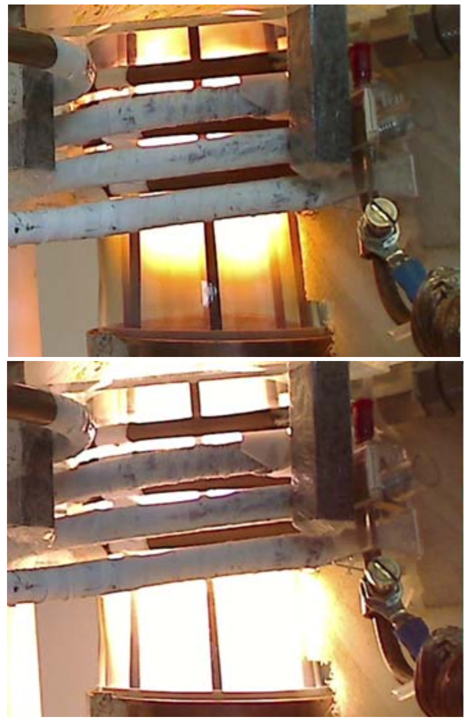
Two synergetic approaches:

=theoretical (typically 1D, full understanding of parameters);

=numerical (simulation, 2D, 3D spatial dimension, 3D + Monte Carlo, 3D + Fokker Planck =6D): the more coordinates included, the much slower the code, the less understanding of parameters.

Nonlinear problem: a small increase of rf power can give much more plasma; this is usually observed in experiment (often related to hysteresis, multistability);

so in a long 3d+ Monte Carlo simulation, if you set unlucky parameters (for power, gas density, and so on), you realize this only at the simulation end.

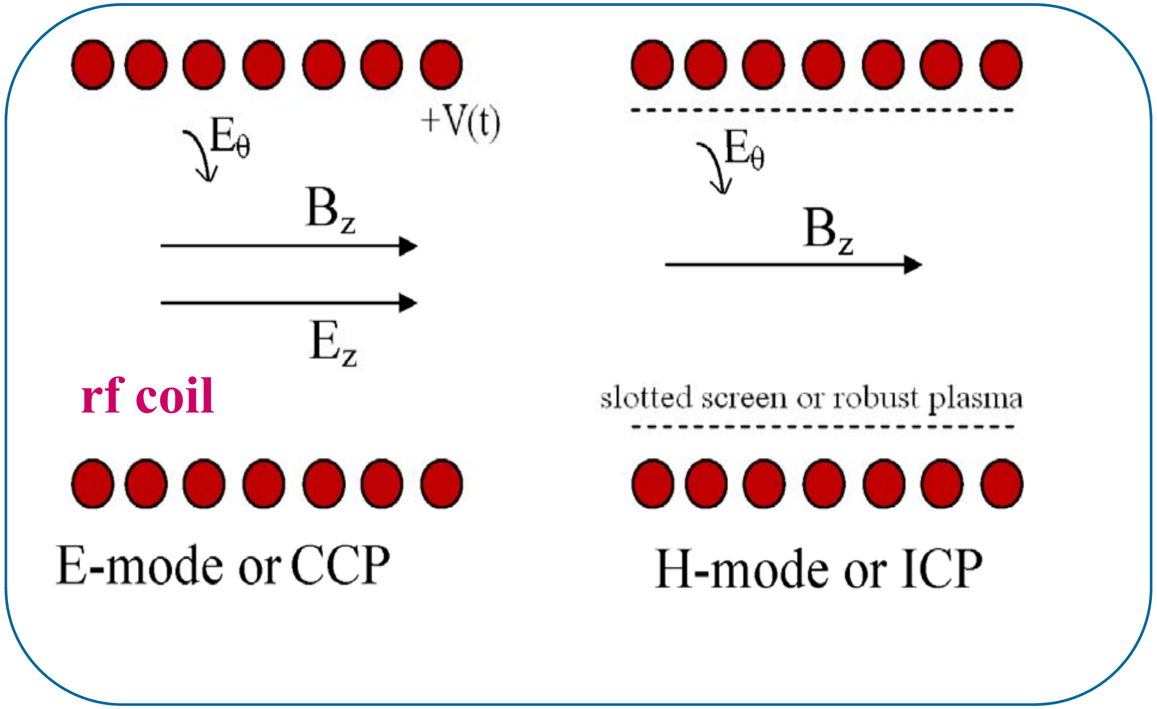


Plasma with total rf power P_t=350 W (above) and 400 W (below); two rf coils and faraday cage partly visible [12]

So here we discuss mainly 2D simulations where the collision are accounted for with theoretical expressions; relevant concepts are skin depth, plasma conductivity, electron temperature and collision frequency.

Moreover, the plasma can couple to rf coil in two modes:

1) Capacitive Coupled Plasma (E-Mode : very low electron density, the axial electric field E_z [13] directly accelerates them, and de-confines them (that, E_z pushes them out of the plasma)



2) Inductive Coupled Plasma (H-mode, dense plasma); axial electric field E_z is suppressed (by a slotted screen or by plasma polarization); the weaker E_ϑ accelerates electrons in multisteps, by stochastic or collision phase mixing, and electron energy distribution is broad (similar to a Maxwellian one).

We restrict to this coupling.

1.4) So our 2D induction model features:

Approximate cylindrical symmetry: each turn k is a ring, driven by rf voltage $2\pi U_k$, to be adjusted tor equal total current in each turn.

Dielectric window allowed, but no ferrite considered (μ_r=1)

Use a local conductivity σ model for the plasma (see later)

Plasma (or a screen) shields electric potential; for an helical coil, only azimuthal part of vector potential remains:

$$\phi \cong 0$$

$$\mathbf{A} \cong \Re\, \hat{\vartheta} A_\vartheta(r,z) \mathrm{e}^{\mathrm{i}\omega t}$$

$\Re$ (real part of) is usually understood in the following (phasor notation) so Maxwell's equations give

$$rA_{\vartheta,zz} + (rA_{\vartheta,r})_{,r} + rQA_\vartheta = \mu_0 \sigma U_k \qquad \text{(2a)*}$$

where a comma means 'partial differentiation' and Q depends on material

$$Q = -r^{-2} - \mathrm{i}\mu_0 \omega \sigma + \varepsilon_r (\omega/c)^2 \qquad \text{(2b)}$$

*Note: equation number (displayed only when later referenced) follows numbering in proceeding paper

2)The conductivity in plasma (mainly due to electrons)

In plasma, rf field strength is not uniform (typically it is decaying, that is the skin effect), and rf includes both magnetic and electric field so electron motion is very complicate, as easily seen in one-particle simulations, also for weak plasma (electron density n_e to zero)

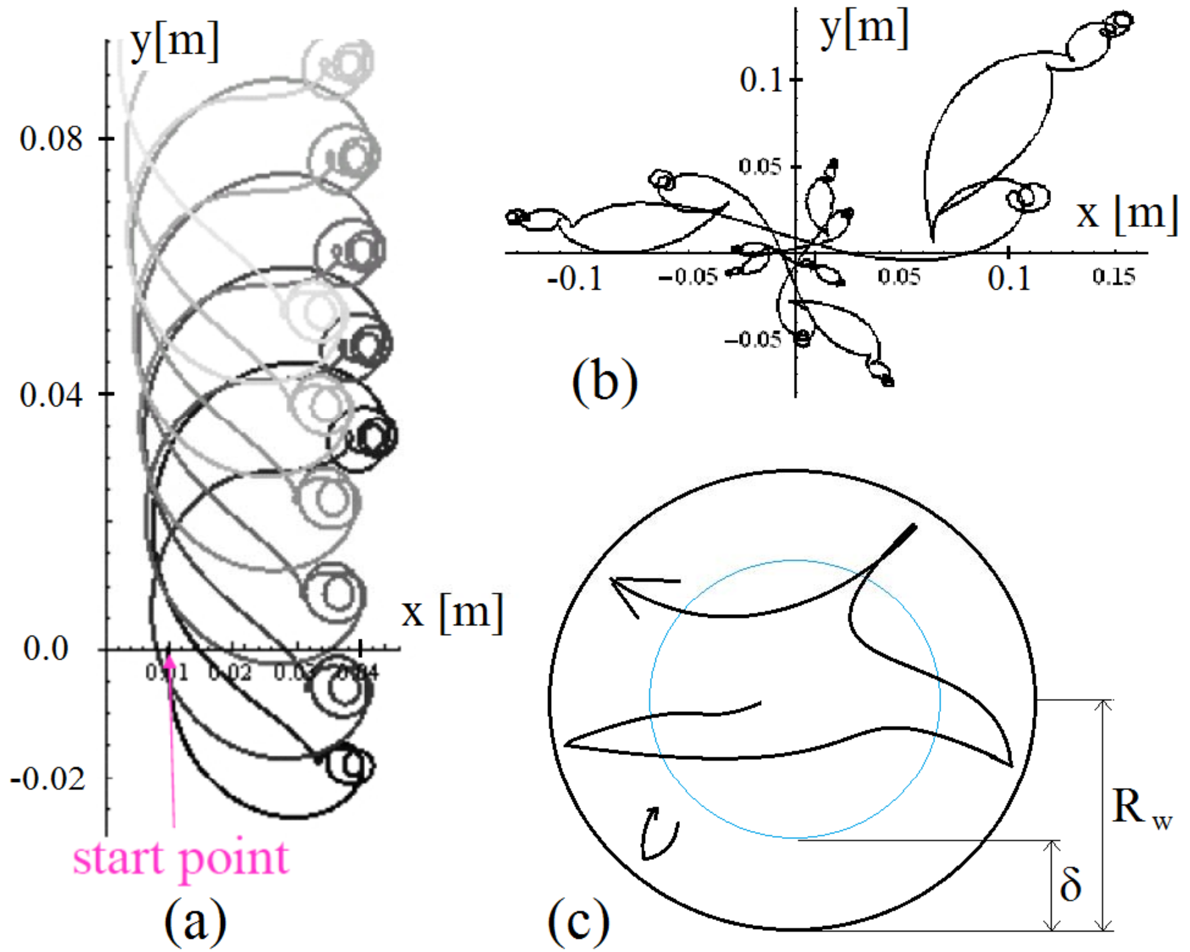


Figure: samples of electron orbits in rf fields in
(a) weak plasma, uniform B_z and E_x
(b) weak plasma, uniform B_z and $E_\vartheta \propto r$
(c) strong plasma, ie skin depth δ smaller then radius R_w

The local conductivity σ model

Strictly speaking conductivity is a nonlocal operator (defined by a functional derivative)

$$\sigma = \check{\delta}\mathbf{j} / \check{\delta}\mathbf{E}$$

A local expression including only gas collision friction or 'collision equivalenced' effect is $\sigma = n_e e^2 / (\nu_c + \mathbf{i}\omega)$ **with** $\nu_c = \nu_m + \nu_s$

ν_m collision frequency due to real collisions with gas or ions;

ν_s stochastic term to fit anything else, like collisions with walls or electron oscillation larger than skin depth

The material function Q is then simply written in term of the solid material permittivity ε_r, (with ω_p=0) or of the effective collision frequency and of the plasma frequency ω_p (with ε_r=1)

$$Q = \varepsilon_r \frac{\omega^2}{c^2} - \frac{1}{r^2} - \frac{\omega_p^2}{c^2} \frac{\mathrm{i}\omega}{\nu_c + \mathrm{i}\omega}$$

Planar approximation and skin depth δ formula

Meaning and effect of eq (2) are more easily in planar approximation

$$y = r \sin\vartheta \ \text{ and } \ x = r\cos\vartheta - R_w \ \text{ with } R_w \to \infty$$

relevant solution of (2) is then the (decaying) wave

$$A_\vartheta \cong A_y = a_1 \exp(\mathrm{i} k_x x) \qquad k_x = k_r - (\mathrm{i}/\delta)$$

where the skin depth is $\delta = -1/\Im k_x$ with $k_x = \sqrt{Q_\infty}$

and Q_∞ is Q for r to ∞ . With some algebra, exact solution gives

$$\delta = \frac{c\sqrt{2(1+\gamma^2)}}{\sqrt{\omega_s^2 - \nu_c^2 + \sqrt{1+\gamma^2}\sqrt{\omega_s^4 + \nu_c^2\omega^2}}}$$

$$\gamma = \nu_c/\omega$$

$$\omega_s^2 = \omega_p^2 - \omega^2$$

In induction plasma $\omega_p \gg \omega, \nu_c$ this approximate as

$$\frac{\delta}{c} \cong \frac{\sqrt{2(1+\gamma^2)}}{\sqrt{1+\sqrt{1+\gamma^2}}} \frac{1}{\omega_s} \qquad (5)$$

3) Unmagnetized plasma: The effective collision frequency

Let us recall that, in some simple case [6] as electron bouncing from a plasma/wall sheath, power absorbption P_w can be calculated from kinetic and nonlocal model.

$$\frac{P_w}{|E_w|^2} = n_e^w \frac{e^2 \delta^2}{m_e v_{th}} I_1(\alpha) \quad , \quad \alpha = \frac{4\omega^2\delta^2}{\pi v_{th}^2}$$

with thermal velocity $v_{th} = (8T_e/\pi m_e)^{1/2}$

Γ is here incomplete gamma function

$$I_1(\alpha) = [(1+\alpha)e^{\alpha}\Gamma(0,\alpha) - 1]/\pi$$

Note that the time electron spend inside rf skin layer is $\tau = 2\,\delta/v_{th}$, so $\omega\tau$ is a dimensionless parameter, as well α is.

The effective coll. frequency is defined such as to obatain the same power absotion which gives

$$\frac{\nu}{1+\nu^2} \cong 2\sqrt{\pi\alpha}\, I_1(\alpha) \qquad \textbf{(8)}$$

This has two solutions for ν, shown as ν^+ or ν^- in the figure

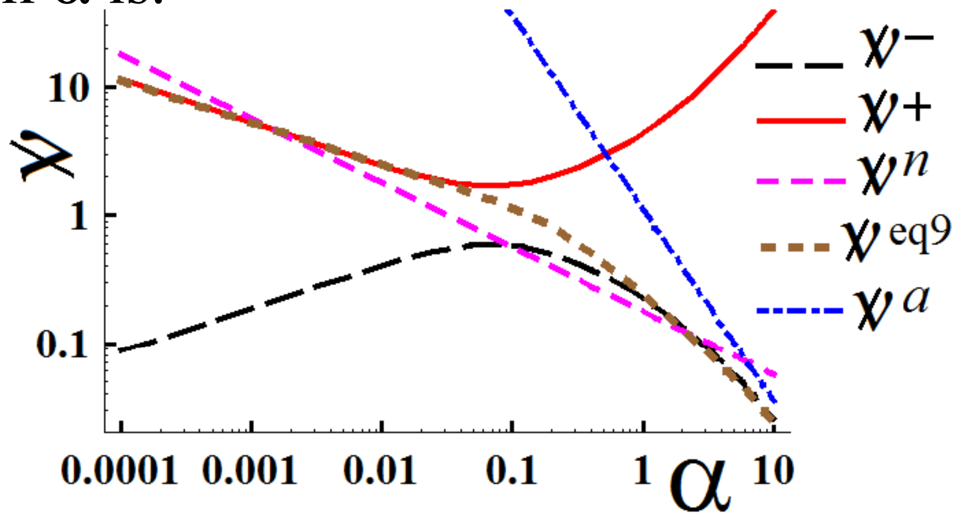


Figure: Plots of ν_c/ω vs α, from eqs. (8) or (9).

What branch?

Accepted limits in literature [6,8,9] are:

For $\alpha \rightarrow \infty$ (that is travel time $\tau >> 2\pi/\omega$), rf averages on many cycles, so lower branch should selected; limit (see previous picture) is

$$\nu^a = 2/\sqrt{\pi\alpha^3}$$

For $\alpha \ll 0.01$ (that is travel time $\tau << 2\pi/\omega$) electron 'collides' with a fast impulse of rf, so upper branch is usually approximated as

$$\nu^n = 1/\sqrt{\pi^3\alpha}$$

Here we state an interpolation including all values of α

$$\nu = \frac{\sqrt{1/\pi}}{\sqrt{2\alpha^2 + \frac{1}{4}\alpha^3 + \alpha\left[2 + \frac{4}{\pi^2}\log^2(c_1/\alpha)^2\right]}}$$

$$c_1 = \mathrm{e}^{-1-\gamma} \cong 0.206$$

$$\gamma \cong 0.577$$

(eq 9)

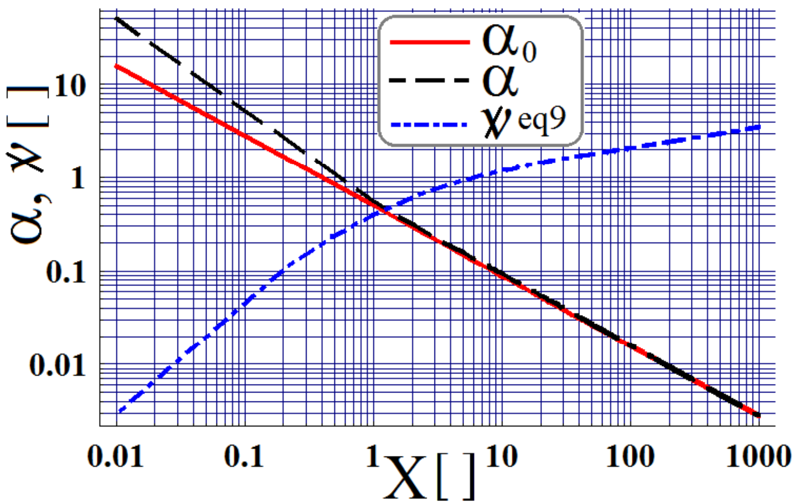
But α and ν_c are related also by (5) or:

$$\alpha = \frac{m_e\omega^2\delta^2}{2T_e} \cong \frac{1}{X}\frac{1+\nu^2}{1+\sqrt{1+\nu^2}} \qquad \textbf{(10)}$$

with a dimensionless parameter X:

$$X \equiv \frac{T_e\omega_s^2}{m_e c^2\omega^2} \cong \frac{n_e^w T_e}{p_0} + O\left(\frac{\omega^2}{\omega_p^2}\right) \qquad p_0 = \frac{m_e^2\omega^2}{\mu_0 e^2}$$

<u>measuring plasma pressure</u> n T_e in units of p_0; Eqs. (9,10) <u>have a universal solution (see graph)</u> for α and ν_c



Plot of ν_c/ω and α vs parameter *X (also some initial estimate α_0 is shown)*

4) Magnetized plasmas and plasma model

We have to consider static magnetic fields B_s, which gives the well known cyclotron frequency Ω_s=e B_s/m_e, and rf magnetic field with amplitude B_f, and similarly Ω_f=e B_f/m_e. We can combine both as

$$\Omega_t^2 = \frac{1}{2}\Omega_f^2 + \Omega_s^2$$

Generalizing Ref. [8] formula, the conductivity is expressed as

$$\sigma = \frac{n_e e^2}{m_e \sqrt{\nu_m^2 + \Omega_t^2}} \left(1 - \frac{i\omega \nu_m}{\nu_m^2 + \Omega_t^2} + O(\nu_m^2) \right) \qquad \omega \ll \nu_m < \Omega_t$$

Simpler models just requires that flows of ions Γ^i and electrons Γ_e originate from ionization rate n_{iz}

$$\operatorname{div} \mathbf{\Gamma}_i = \operatorname{div} \mathbf{\Gamma}_e = n_g n_e K_{iz}(T_e) = n_{iz}$$

and plasma heat diffusion balances with electromagnetic heat P_h and energy loss in ionization

$$-\nabla(K_e \nabla T_e) = P_h - n_e n_g K_{iz} \mathscr{E}_{iz}$$

$$P_h = \frac{1}{2}\Re(j_\vartheta^* E_\vartheta)$$

where K_e is thermal conducitivity, $K_{iz}(T_e)$ is the ionization costant (see graph for n_{iz}) and $\mathscr{E}_{iz}$ is the energy loss per ionization pair (see graph)

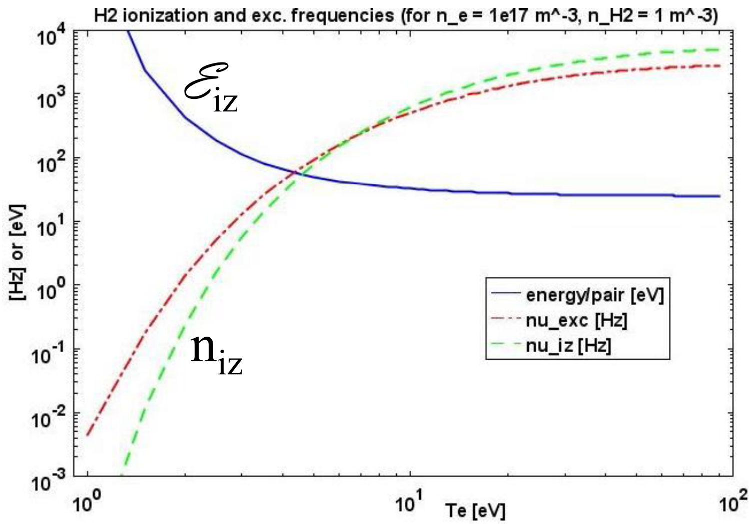


Figure: $\mathscr{E}_c$ the energy lost per pair (e ion+, effective values for H_2) produced vs the plasma electron temperature T_e; note its peak for T_e<3 eV. Reason is that excitation rate is there much greater than ionization rate, as shown

5) Work flow of a typical multiphysics simulation

There are two good reasons for iterative solving of previous model:

1) Some variables (as n_e or T_e) are real valued, some are complex (magnetic potential phasor), so n_e and T_e must be kept real against rounding error effects

2) The problem is nonlinear (it may have many solutions in principle), so the user has to give an adequate initial guess, which is easier to imagine for real variables alone.

As a practical fact, computer RAM is limited (not a TB yet): so in our code solution is also performed in 2D with preliminary averaging of the static magnetic field (which has a 3D structure, with strong multipoles).

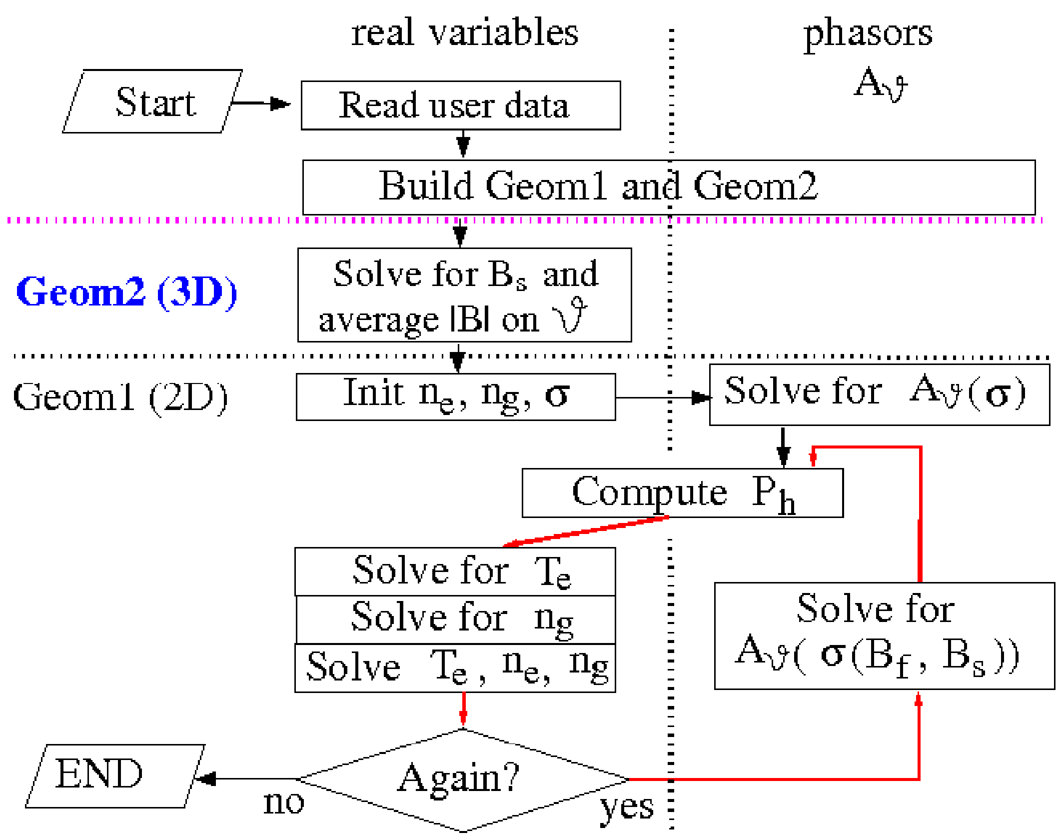


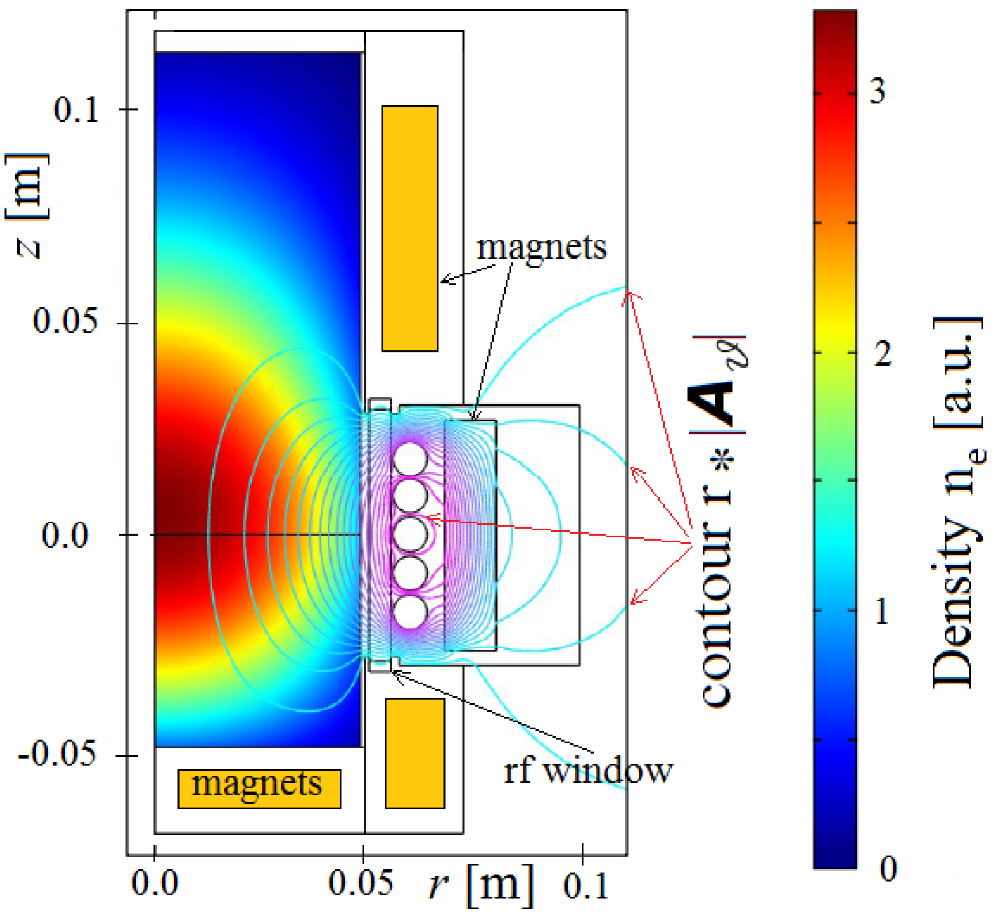
Figure 5. Major steps of numerical simulations: real and complex variables must be solved in separate steps.

5.2) Results

Beware: Any correct model of ionization and rf absorption in plasma typically includes a possible instability: the more electron are produced the more rf power can be adsorbed which gives even more electrons, provided gas density n_g and coil current I_0 are kept constant. Stabilization is more easily built in the model by adjusting gas density so to have a reasonable plasma density at a given point (set by experience or as experimental input data)

Once model has converged, the density typically peaks on source axis, where plasma confinement is better (so more plasma accumulates)

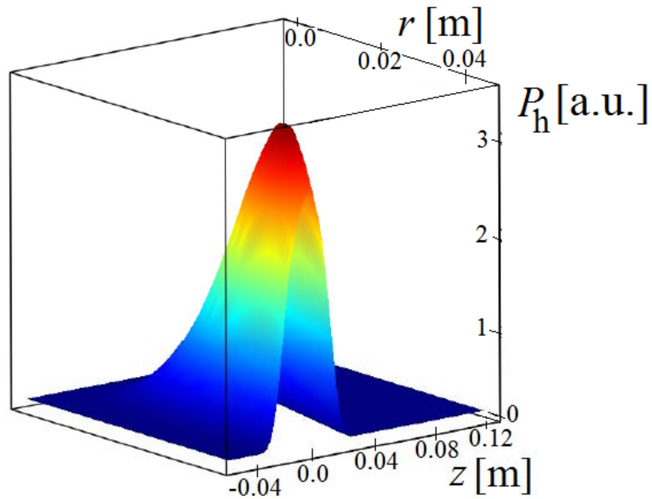
Similarly the induction rf filter peaks on the rf coil; it is possible to define pseudo flux lines of rf magnetic field, as the contour level of r $|A_\vartheta|$, where the absolute value is needed for the phasor rf field; in static limit, these contours gives the usual flux lines.



Plasma density n_e and 'pseudo-flux-lines' of rf magnetic field (that is, level curve of $r|A_\vartheta|$)

Results (continues)

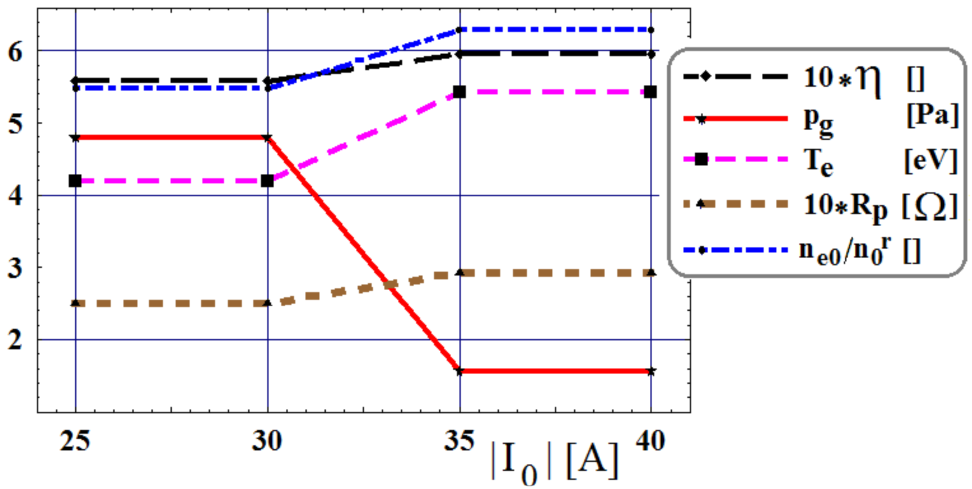
But rf power deposition P_h peaks on coil middle plane, somewhat inside rf window radius, at some $r=r_M$, since it is a product of electron density and rf electric field (and conductivity). The actual value of R_M so predicted is helpful for comparison to several circuit models [9].



Plasma heating power density P_h.

The total rf power includes heating in the plasma and loss in the rf coil, in the vacuum chamber and so on-

Finally, the gas density we need to sustain a stable plasma is found to drastically depends on coil current I_0, with a threshold. This well mimics the experimental fact that we need to exceed some power for induction plasma turn on, usually also by decreasing pressure.



Preliminary simulation results (see legend for units) vs rf current |I0|; here p_g is gas pressure (NTP), T_e is central electron temperature, n_{e0} central gas density, $n_e^r = 10^{17}$ m^{-3}, R_p equivalent resistance of coil+plasma system, η efficiency (ratio of power in plasma and total rf power)

6) CONCLUSION

Induction heating of plasma is a typical nonlinear problem, with gas ionization rate and rf power absorption in positive feedback. Stability is obtained (both in the experiment and in the modeling) by the limited amount of rf power and gas available. While a detailed calculation of each electron trajectory (possibly depending of its phase to radiofrequency) is clearly too long especially for ion source design, a vast literature has developed useful approximation to this problem, introducing the so called stochastic effect, with several formulas in limiting cases; here a formula encompassing all of them was given. For magnetized plasmas, a simple model is detailed; its solution well reproduce observed trends for gas density, equivalent plasma resistance and plasma luminosity.

Acknowledgements

Work supported by experiment Ion2neutral (group 5 of INFN, Istituto Nazionale di Fisica Nucleare) and project INFN-E (Energy applications).

THANK YOU FOR ATTENTION

(see bibliography next slide)

References

[1] M. A. Lieberman and V. A. Godyak, “From Fermi Acceleration to Collisionless Discharge Heating,” IEEE Trans. Pla. Sci., 26, 1998, pp. 955–986, doi:10.1109/27.700878.

[2] M. A. Lieberman and A. J. Lichtenberg, “Principles of Plasma Discharges and Materials Processing,” New York, Wiley, 2005 (2nd ed.), doi:10.1002/0471724254.

[3] I. D. Kaganovich, V. I. Kolobov, and L. D. Tsendin, “Stochastic electron heating in bounded radio-frequency plasmas” Appl. Phys. Lett., 69, 1996, pp. 3818–3820, doi:10.1063/1.117115.

[4] G. Torrisi, D. Mascali, A. Galatà, et al., “Plasma heating and innovative microwave launching in ECRIS: models and experiments,” J Instrum, 14, 2019, C01004, and references therewithin, doi:10.1088/1748-0221/14/01/C01004.

[5] M. Cavenago and C. T. Iatrou, “Studies on microwave coupling into the electron cyclotron resonance ion source Alice,” Rev. Sci. Instrum., 65, 1994, pp 1122– 1124, doi:10.1063/1.1145084.

[6] V. Vahedi, M. A. Lieberman, G. DiPeso, T. D. Rognlien, and D. Hewett, “Analytic model of power deposition in inductively coupled plasma sources,” J. Appl. Phys., 78, 1995, pp. 1446–1458, doi:10.1063/1.360723.

[7] M. Cazzador, M. Cavenago, G. Serianni, and P. Veltri, “Semi-analytical modeling of the NIO1 source,” AIP Conf. Proc., 1655, 2015, 020014, doi:10.1063/1.4916423.

[8] M. Tuszewski, “Inductive electron heating revisited,” Phys. Plasmas, 4, 1997, pp. 1922–1928, doi:10.1063/1.872335.

[9] P. Jain, M. Recchia, M. Cavenago, U. Fantz, E. Gaio, W. Kraus, A. Maistrello and P. Veltri, “Evaluation of power transfer efficiency for a high power inductively coupled radiofrequency hydrogen ion source,” Plasma Phys. Control. Fusion, 60, 2018, 045007, doi:10.1088/1361-6587/aaab19.

[10] D. Rauner, S. Briefi and U. Fantz, “RF power transfer efficiency of inductively coupled low pressure H2 and D2 discharges,” Plasma Sources Sci. Technol.,26, 2017, 095004, doi:10.1088/1361-6595/aa8685.

[11] F. F. Chen, “Nonlinear Effects and Anomalous Transport in RF Plasmas,” IEEE Trans. Pla. Sci., 34, 2006, pp. 718–727, doi:10.1109/TPS.2006.874851.

[12] M. Cavenago and S. Petrenko, “Models of radiofrequency coupling for negative ion sources,” Rev. Sci. Instrum., 83, 2012, 02B503, doi:10.1063/1.3670601.

[13] K. Henjes. “Electric and magnetic fields in solenoidal coils from statics to MHz Frequencies”, J. Appl. Phys., 79 (1996) 21-29.

[14] S. Lee, Y. Lee and Yu I., “Electric Field in Solenoids”, Jpn. J. Appl. Phys., 44 (2005) 5244-5248