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Determination of Exact Ray Paths by Bidirectional Ray-Tracing

Mehmet Mert Taygur(1) and Thomas F. Eibert(1)
(1) Technical University of Munich, Department of Electrical and Computer Engineering
Chair of High-Frequency Engineering, Munich, Germany

Abstract

The detrimental effects of phase errors due to improperly selected reception spheres in unidirectional ray-tracing are demonstrated. The phase errors, which arise from incorrect ray path length calculations, are diminished by a bidirectional ray-tracing approach in combination with an asymptotic approximation. The evaluation of the reciprocity integral in the conventional bidirectional ray-tracing approach is reduced into an algebraic expression, which involves the values of the integrand and its derivatives at a stationary phase point. The stationary phase point is found by utilizing the Fermat principle of least time where the trajectory of the ray path with shortest length is calculated. Thus, the path length calculation is performed in a much more accurate manner. Numerical results show the advantages of the approach.

1 Introduction

Ray-tracing has recently become the standard simulation tool for many applications in electromagnetics, as the emergence of massively parallel computation paradigms on Graphics Processing Units (GPUs) has enabled fast calculations with decent accuracy [1, 2]. In general, the main goal of the simulation is to compute the antenna transfer function for a transmitter and a receiver antenna where the geometry is usually much larger than the wavelength. The simulations rely on high-frequency approximations such as Geometrical Optics and Uniform Theory of Diffraction (GO-UTD), and a Shooting and Bouncing Rays (SBR) algorithm is commonly utilized for identifying the feasible ray paths [3, 4]. In SBR-based simulations, usually a large number of rays are launched from the transmitter site and traced throughout the geometry. The rays, which intersect with the so-called reception sphere at the receiver site, are processed further to calculate the field expressions and the transfer function. Such an approach has usually considerable benefits in complex scenarios, compared to the alternatives such as the image method.

Despite their numerous advantages, SBR-based approaches in the traditional unidirectional ray-tracing may lead to significant accuracy issues under certain circumstances. More specifically, the size of the reception spheres and the number of the ray launches should be simultaneously fine-tuned

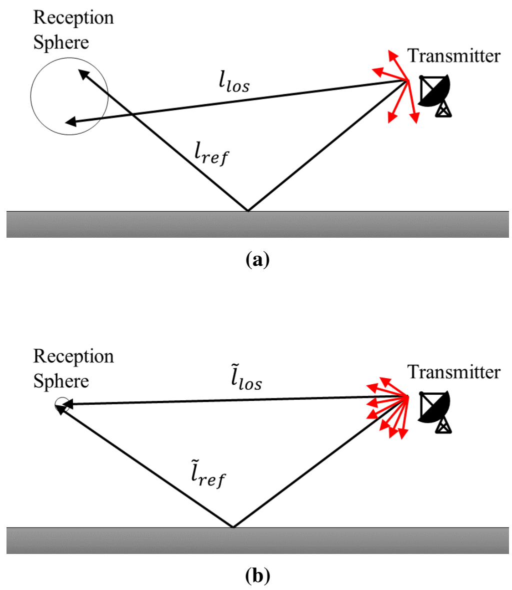

Figure 1. Illustration of the ray path length differences for a small/large number of ray launches and accordingly chosen reception sphere sizes. The errors are likely to be more drastic for l_{los} and l_{ref} than $\tilde{l}_{los}$ and $\tilde{l}_{ref}$

in order to prevent incorrect ray contributions as well as ray misses (i.e., a correct ray path does not hit the sphere and remains undetected). Such an objective usually implies that the number of ray launches is large and the size of the reception sphere is small. Increasing the number of ray launches may not always be practical though, due to the growth in the computation time, which might be exponential if diffractions are also involved in the simulations [5]. Therefore, utilizing relatively large spheres with a small number of ray launches might be convenient in many practical cases. Nevertheless, choosing the sphere size inappropriately may have further implications in the millimeter-wave regime and beyond, as the lengths of the ray paths might be computed inaccurately. Hence, the phase of multipath contributions may become erroneous if the reception spheres are large compared to the wavelength. Consequently, the accuracy of the transfer function may deteriorate.

The implications of improper parameter selections regarding incorrect ray contributions and misses have been previously acknowledged in various studies [6]. However, the phase errors which arise in higher frequencies have not been adequately addressed in the literature. Therefore, in this study, the accuracy issues related to the conventional reception sphere approach are considered where phase errors in the millimeter-wave regime are demonstrated. A remedy for the issue is provided by the bidirectional ray-tracing method. The integration algorithm, which was utilized in the traditional bidirectional ray-tracing approach [6], has been changed to treat millimeter-wave scenarios in a more efficient way. The approach is based on the asymptotic expansion of the reciprocity integral and utilizes the Fermat principle of least time where the properties of the exact rays (i.e., the rays which precisely hit the receiver location) are predicted on an interaction surface. Thus, the path length computation can be carried out much more accurately. A detailed description of the method is presented in [7].

2 Identifying the Exact Ray Paths

Identifying the exact ray paths in a ray-tracing simulation can be considered as a minimization problem in accordance with the Fermat principle of least time, which states that a ray follows the shortest possible path between two points [8]. Based on this information, we now consider a surface Ψ, which separates two antennas, namely, A and B, in a free-space environment. Assuming that a bidirectional ray-tracing procedure has been applied in this scenario, the properties of the wavefronts from both A and B are known on Ψ. Using the knowledge about the wavefronts it is possible to compute the lengths of the paths, which link A and B, on the surface Ψ. The point, where the path length is minimal, can be considered as a stationary point and it is essentially the intersection point of the feasible ray path and the surface Ψ. Note that in a case with scatterers present in the environment, multiple stationary phase points may occur on Ψ. Here, the surface Ψ is assumed to be large enough to sufficiently separate A and B, though closed surface configurations can also be used in many cases. The minimization process can be considered as a convex optimization problem in general, therefore, many different solution methods are available. A minimization algorithm based on a line-search method is described in [7].

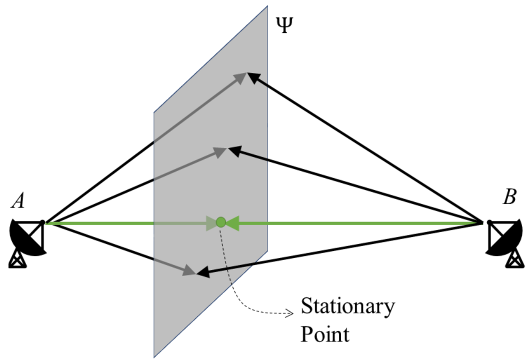

Figure 2. Illustration of stationary point identification approach. Among the ray pairs which emanate from A and B, those which are marked as green yield the path with shortest length. The intersection point of these rays with the surface Ψ is the stationary point.

3 Calculation of Antenna Transfer Function

Once the trajectory of the exact ray paths have been identified on the interaction surface Ψ, the transfer function can be computed by evaluating the reciprocity integral. As stated previously, the evaluation of the reciprocity integral is based on a much more efficient technique, compared to that considered in [6], and helps to diminish the computation time at high frequencies.

3.1 Reciprocity Integral in Oscillatory Integral Form

Recalling the previously considered antenna configuration, let A be a receiver and B be a transmitter. The transfer function H for this scenario can be written as

$$H = \frac{-1}{I_A V_B^{gen}} \oiint_{\Psi} [(\mathbf{H}_A \times \mathbf{E}_B) - (\mathbf{H}_B \times \mathbf{E}_A)] \cdot d\mathbf{S}, \quad (1)$$

where I_A is the port current at antenna A, V_B^{gen} is the generator voltage at antenna B, and $\mathbf{E}_A$, $\mathbf{E}_B$, $\mathbf{H}_A$, $\mathbf{H}_B$ are electric and magnetic fields from A and B with a suppressed time dependency of $e^{j\omega t}$, respectively. The surface integral term in (1) can be expressed in an oscillatory integral form, which is given by

$$\oiint_{\Psi} [(\mathbf{H}_A \times \mathbf{E}_B) - (\mathbf{H}_B \times \mathbf{E}_A)] \cdot d\mathbf{S} = \iint_{\Psi} f(\mathbf{r}) e^{jkg(\mathbf{r})} dS, \quad (2)$$

with

$$\alpha(\mathbf{r}) = \left[(\mathbf{H}_A(\mathbf{r}) \times \mathbf{E}_B(\mathbf{r})) - (\mathbf{H}_B(\mathbf{r}) \times \mathbf{E}_A(\mathbf{r})) \right] \cdot \hat{\mathbf{n}},$$
$$f(\mathbf{r}) = ||\alpha(\mathbf{r})||, \; g(\mathbf{r}) = \frac{\arg(\alpha(\mathbf{r}))}{k}, \quad (3)$$

where f and g are the magnitude and phase functions, respectively. Various numerical integration methods can be employed to evaluate such an integral, however, many conventional techniques might be extremely time consuming if the operating frequency is at or beyond millimeter-wave regime [9, 10]. In order to address this issue, an asymptotic approach is used in this study.

3.2 Asymptotic Expansion for the Oscillatory Integral

The result of the oscillatory integral given in Eq. (2) is strongly dependent on the values of f and g as well as their derivatives at certain points lying on Ψ. If there is a point $\mathbf{r}_0 \in \Psi$ such that $\nabla g(\mathbf{r}_0) = 0$, it is a stationary phase point and the integral result can be given by [11]

$$I \sim \frac{2\pi f(\mathbf{r}_0)}{k\sqrt{\det(\text{Hess}(g(\mathbf{r}_0)))}} e^{jkg(\mathbf{r}_0)+j\pi/4} + \mathscr{O}(k^{-1}), \quad (4)$$

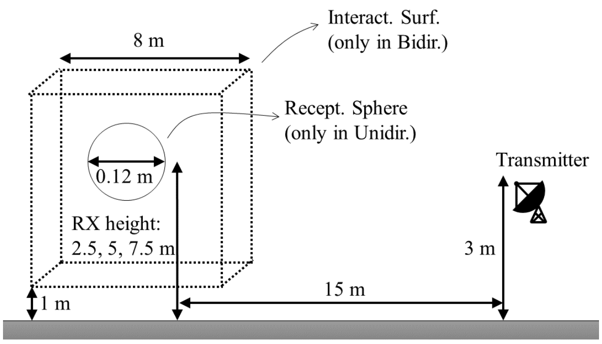

Figure 3. Two-ray ground reflection scenario illustration

as $k \to \infty$. Note that the condition $\nabla g(\mathbf{r}_0) = 0$ indicates that the corresponding path length is minimal. As the phase function g is mostly dependent on the lengths of the ray paths, $\mathbf{r}_0$ can also be associated with the definition of a stationary point which was previously given. In other words, $\mathbf{r}_0$ denotes the point where the feasible ray (having minimum path length) intersects with Ψ. The error term $\mathscr{O}(k^{-1})$ indicates that the accuracy of this technique typically increases with the frequency. The propagation direction vectors for the fields originating from A and B have opposite directions at $\mathbf{r}_0$ in general. This provides a verification opportunity for the minimization problem, i.e., the validity of the minimum can be confirmed by checking the inner product of the propagation direction vectors, which should be equal to -1 (assuming that they are normalized).

In certain scenarios, where $\nabla g(\mathbf{r}_0) = 0$ may not be satisfied, the asymptotic expansion given in Eq. (4) is no longer valid. In such cases, the integral evaluation can be performed according to the method given in [6].

4 Numerical Results

A two-ray ground reflection scenario, which is illustrated in Fig. 3, is simulated for 5 different frequencies from 5 GHz to 25 GHz. The path gain for three different receiver locations is investigated and the deviation from the reference is analyzed. The ground plane is assumed to be PEC. In the unidirectional ray-tracing simulations, a reception sphere with a radius of 0.12 m (corresponding to 2λ distance at 5 GHz) is utilized whereas in the bidirectional ray-tracing case, a cubic interaction surface, which encloses the receiver and has a side length of 8 m, is used. Note that the size of the interaction surface is much larger than the reception sphere, and the electrical size of both objects increases as the frequency increases. In both cases, the same number of ray launches were considered, which is 30 000. The results are shown in Fig. 4.

The results indicate that the phase errors are indeed encountered if a fixed-size reception sphere is utilized under varying frequency conditions. Even though the error does not increase monotonically, deviations are likely to occur if the reception sphere is electrically large. Note that the number

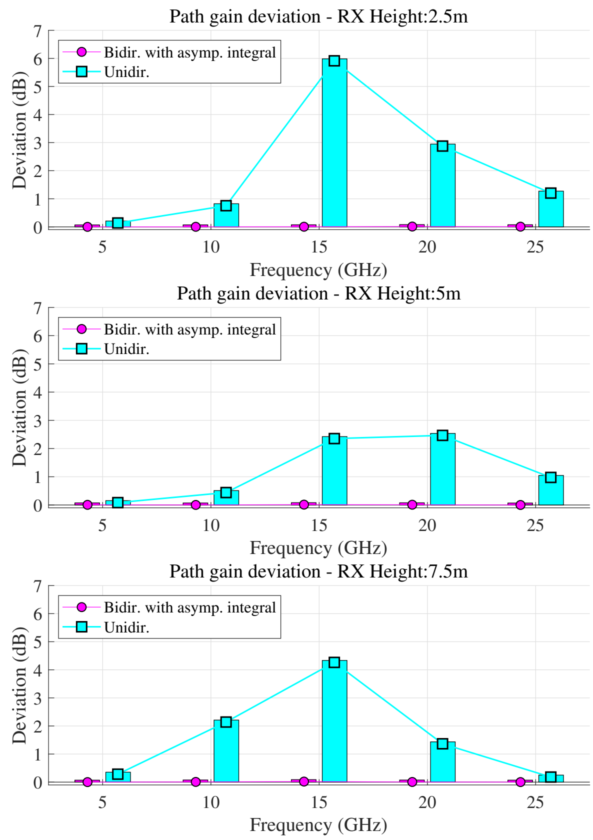

Figure 4. Two-ray ground reflection simulation results

of the multipaths is limited to 2 here, i.e., line-of-sight path and ground reflection path, but it can be argued that the error may grow even further if more scatterers, hence more rays, exist in the scenario. On the other hand, the bidirectional ray-tracing based on asymptotic integration does not exhibit such a deviation, even at higher frequencies. Thus, it can be stated that the proposed method yields better accuracy than the conventional unidirectional ray-tracing for millimeter-wave regime, when the number of ray launches are equal. The reception sphere size in unidirectional ray-tracing should ideally be adjusted with respect to the wavelength, however, this may not always be feasible since the number of ray launches will be accordingly large. For such situations, the proposed method is a viable alternative.

5 Conclusion

The phase errors, which emerge at millimeter-wave unidirectional ray-tracing simulations, were characterized where the utilized reception sphere size was kept constant under varying frequencies. A solution of the problem was provided by a bidirectional ray-tracing method based on asymptotic expansion of the involved reciprocity integral. The solution of the integral was given by the utilization of a stationary phase point, which lies on the exact ray path connecting the receiver and the transmitter. Using the Fermat principle of least time, the stationary phase point can be obtained on an interaction surface and also the integral can be evaluated by a single algebraic expression. The numerical results have shown that phase errors are encountered in unidirectional ray-tracing simulations when the sphere radius is electrically large, though, the error may not always increase with frequency. The proposed approach exhibits a much better performance as the deviation from the reference is very small.

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